



Multi-scale
approaches

Modeling

Aim

Who am I

Multi-scale

Ab-initio

MD

Monte-Carlo

Full Field

CNGTs

DDD

FEM

Conclusions

Multi-scale approaches in Material Science

III-ème école d'été "Modélisation des Matériaux" -
Banyuls-sur-Mer 19-23 Août 2024

Michel Perez

Université de Lyon - INSA Lyon - MATEIS - UMR CNRS 5512



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Merci!



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Modeling... what for?



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What it is and what it is not

✓ A warm up for this summer school

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- ✗ A complete panorama on modelling scales

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- ✓ A climbing up of the modelling scales (to the top?)
- ✗ An ad-free presentation

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Michel Perez: CV in 5 lignes Michel.Perez@insa-lyon.fr

- INSA: 1996
- PhD at INP Grenoble on levitation
- Assistant professor (2001)
- Teaching Physics, Thermodynamics and Material Science
- Research in material science: multi-scale modelling

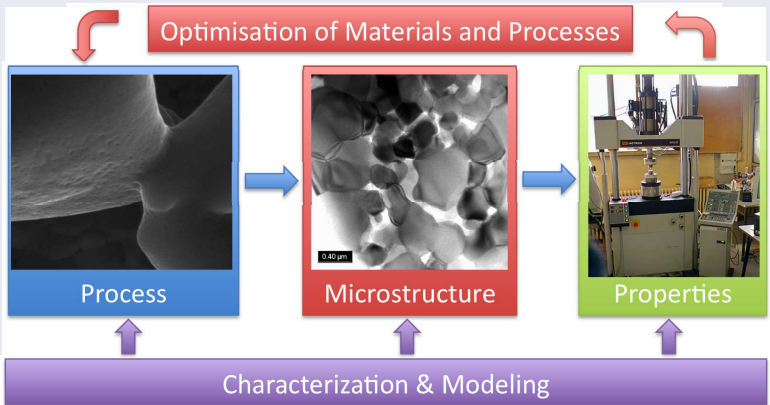
<http://michel.perez.net.free.fr/>

INSA Lyon: MATEIS

MATerials Engineering and Sciences

Multi-scale
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Strategy of the group



Multi-scale approach

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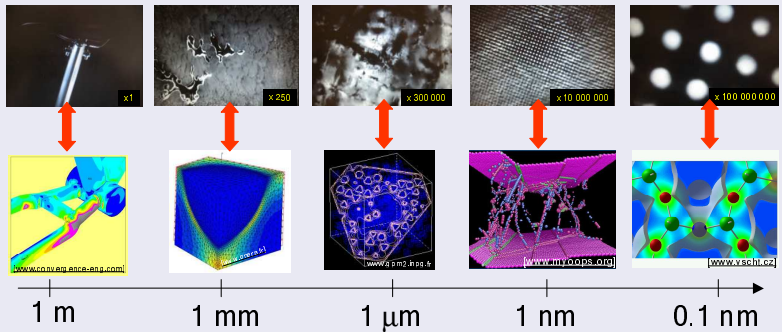
A video to warm up



Multi-scale approach

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Different scales in material science

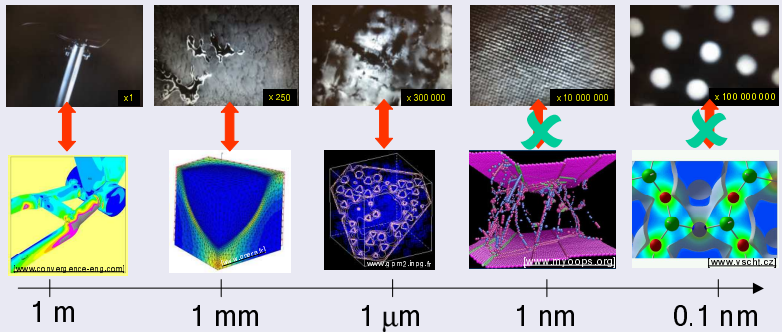


- Classical scheme: experiment $\leftrightarrow$ modelling

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Different scales in material science

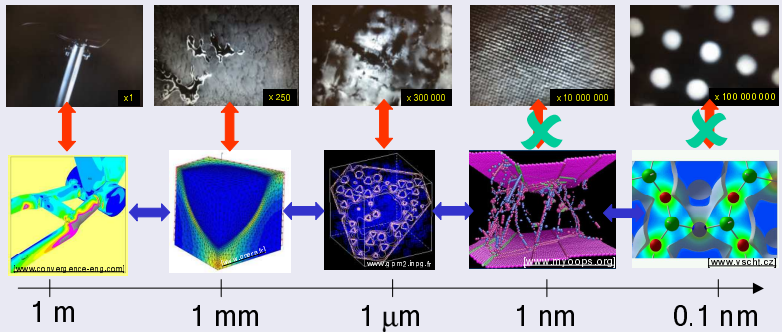


- Not always possible!

Multi-scale approach

Multi-scale
approaches

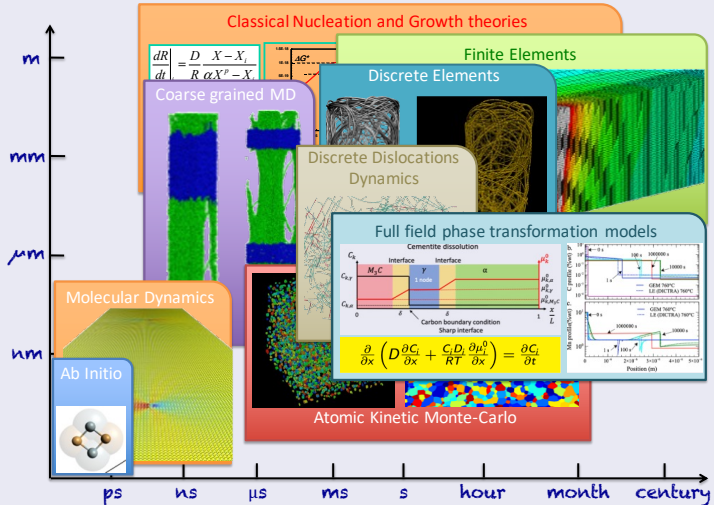
Different scales in material science



- Aim connect modelling at different scales

Multi-scale approach

Different scales in material science



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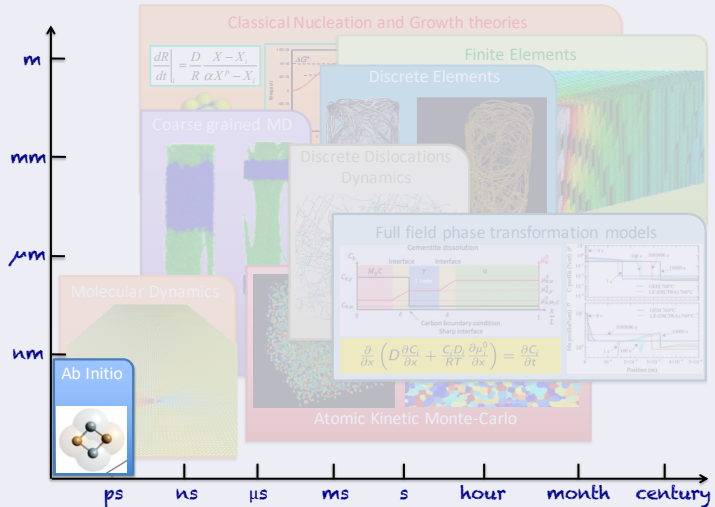
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A journey into the quantum world

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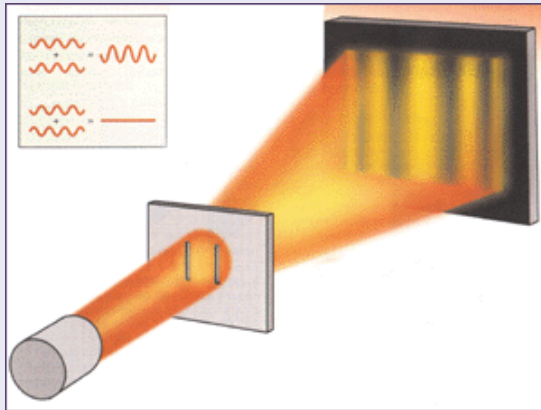
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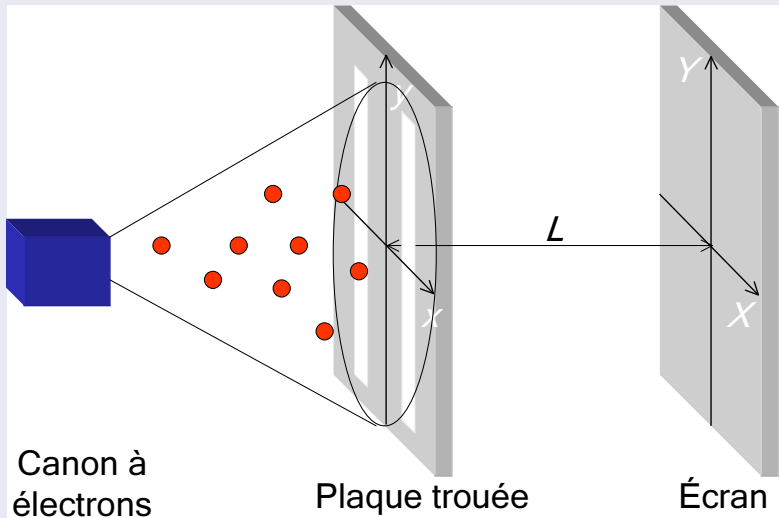
Back to a very simple experiment: Young's double slit (1801)



- Light = waves
- What about matter?

A journey into the quantum world

More complex: with electrons (1961)



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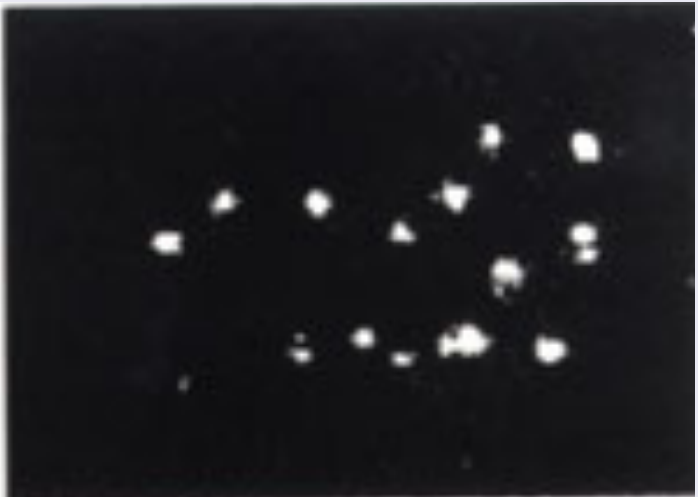
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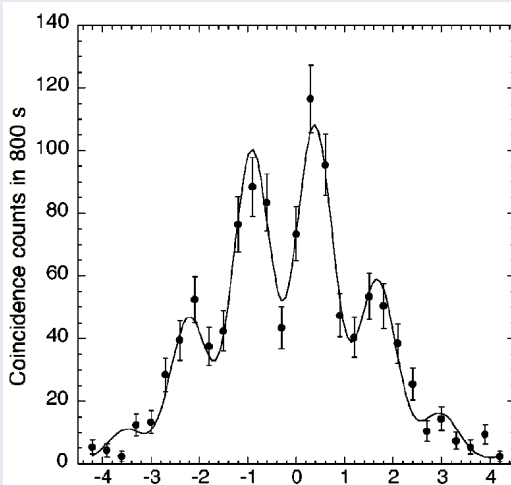
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Double-slit quantum eraser, S. P. Walborn, M. O. Terra Cunha, S. Padua,¹ and C. H. Monken, *PHYS. REV. A* 65(2002)

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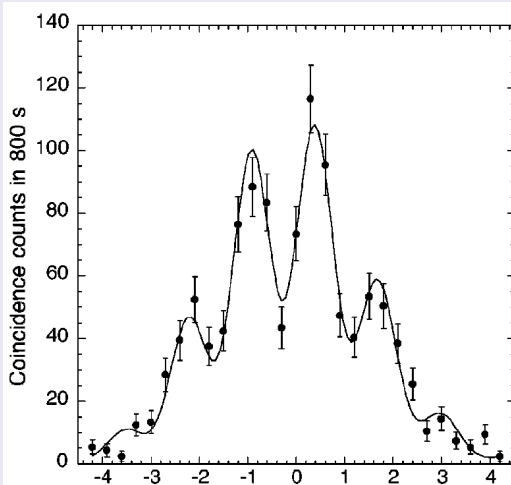
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Even more complex: with a detector (2002)



Double-slit quantum eraser, S. P. Walborn, M. O. Terra Cunha, S. Padua,¹ and C. H. Monken, *PHYS. REV. A* 65(2002)

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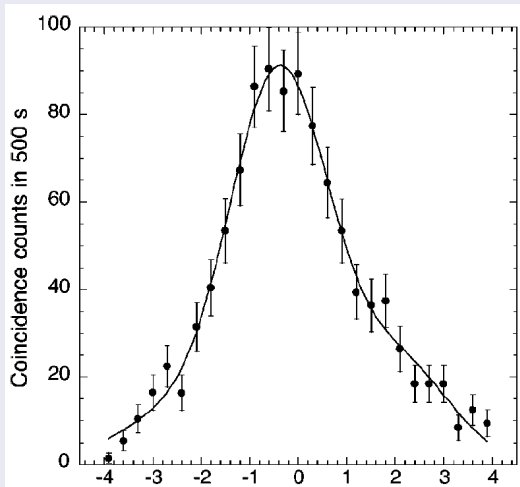
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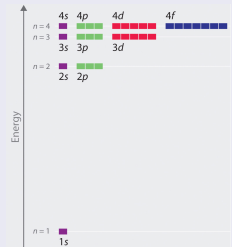
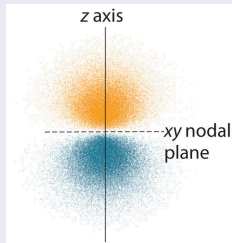
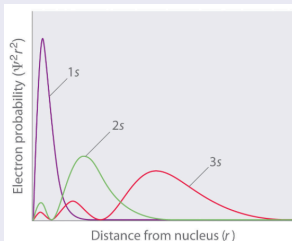
Multi-scale
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Hydrogen atom

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}) + V(|\mathbf{r}|) \psi(\mathbf{r}) = E \psi(\mathbf{r})$$

- $V(|\mathbf{r}|) = e^2/|\mathbf{r}|$
- Laplacian in spherical coordinates (r, θ, ϕ)
- $\psi(\mathbf{r}) = R(r)P(\theta)u(\phi)$

⇒ 3 set of eq. and 3 quantum numbers: n, l, m



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General case

$$\left[-\frac{\hbar^2}{2m} \sum_i \nabla^2 + \sum_i V(\mathbf{r}_i) + \sum_i \sum_j U(\mathbf{r}_i, \mathbf{r}_j) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

⇒ hopeless !

DFT: a path towards simplification

- Find a simpler problem with same solution !
- $E_{\text{exact}}(\mathbf{R}_i, \mathbf{r}_n) \rightarrow E_{\text{approx}}(\mathbf{R}_i, \rho(\mathbf{r}))$

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + \int \frac{e^2 \rho(\mathbf{r}') d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} + V_{\text{XC}}(\mathbf{r}) \right] \psi_i(\mathbf{r}) = \epsilon_i \psi_i(\mathbf{r})$$

Local Density Approximation (exchange correlation energy)

- $V_{\text{XC}}(\mathbf{r}) = V_{\text{XC}}^{\text{electron gas}}(\rho(\mathbf{r}))$

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DFT: practical Implementation

1 Guess initial value of $\rho(\mathbf{r})$



$$2 \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + \int \frac{e^2 \rho(\mathbf{r}') d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} + V_{\text{XC}}(\mathbf{r}) \right] \psi_i(\mathbf{r}) = \epsilon_i \psi_i(\mathbf{r})$$



3 $\psi_i(\mathbf{r})$



$$4 \rho(\mathbf{r}) = \sum_i |\psi_i(\mathbf{r})|^2$$



5 loop to 1 until convergence

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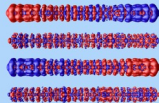
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Conclusions



Andrés Saúl

Département Théorie et
Simulation Numérique
CINaM/CNRS (Marseille,
FRANCE)



COORDINATEUR TDM

Théorie et Simulation
Numérique

RESPONSABLE : ANDRÉS SAÚL

It will be a slow discussion about
concepts that one must understand and methods that one must manage
when one is interested on the electronic properties of
systems composed of atomic nuclei and electrons (molecules, solids, crystals, ...)



- Entrée
 - The hydrogen atom to remember atomic orbitals
 - Multi-electron atoms to discuss the formation of shells and screening
 - A systems with two atoms and two electrons to discuss bonding
- Plat
 - Many atoms and many electrons
 - Methods and approximations neglecting the electron-electron interactions
 - Methods and approximations taking into account the electron-electron interactions
- Dessert
 - La "surprise du chef"

Structure électronique des
matériaux
➢ Lundi 19 Août 11h-13h
➢ Mardi 20 Août: 08h00-10h00

Application

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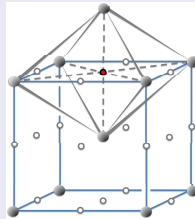
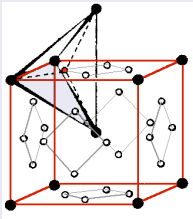
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Conclusions

Find interatomic potential

- Goal: Fe-C interaction potential for Molecular Dynamics
- Two reference configurations: C in octa and tetra sites



- Correct energy barrier for diffusion: 0.815 eV

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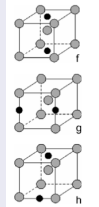
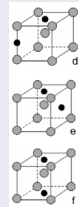
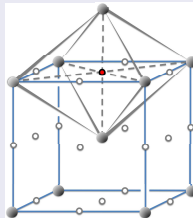
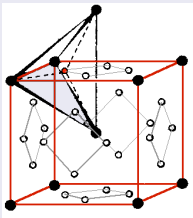
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Conclusions

Find interatomic potential

- Goal: Fe-C interaction potential for Molecular Dynamics
- Two reference configurations: C in octa and tetra sites



- Correct energy barrier for diffusion: 0.815 eV
- Test on many configurations (C, Fe, Va)

Tabulated potential for Molecular Dynamics



[Becquart et al, Comp. Mat. Sc. **40** (2007)]

Atomic Kinetic Monte-Carlo

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**Petr Grigorev, Arnaud
Allera et Martin Uhrin**
MATEIS, IRSN et INP-UGA



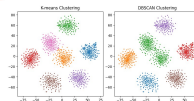
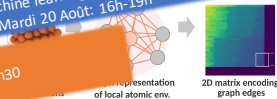
- 1.1 Python ecosystem
- 2.1 Neighbors maps
- 2.2 Structural analysis (ML)

Ecosystème python
➢ Lundi 19 Août: 16h-19h

Machine learning
➢ Mardi 20 Août: 16h-19h

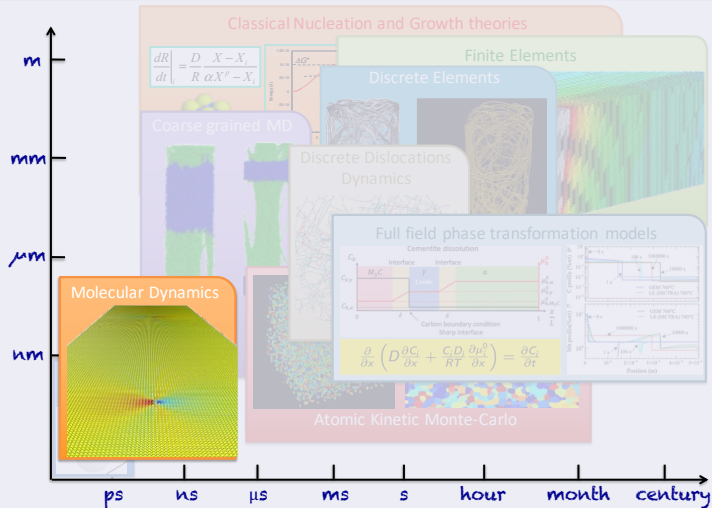
Machine Learning
➢ Mardi 20 Août: 10h30-12h30
➢ Jeudi 22 Août: 8h-10h

tinyurl.com/ModMat2024



Multi-scale approach

Different scales in material science



Principle

- Second Newton's law

$$m_i \frac{d\mathbf{v}_i}{dt} = \mathbf{f}_i$$

- Pair interactions: $\mathbf{f}_i = \sum_j \mathbf{f}_{ij} = \sum_j \mathbf{grad} U_{ij}$
- Integration in various "ensembles": NVE, NVT, NPT...

EAM potential (embedded atom method)

- Pair interaction energies AND electronic density

$$E_{tot} = \frac{1}{2} \sum_i \sum_{j \neq i} U_{ij}(\mathbf{r}_{ij}) + \sum_i F_i \left(\sum_{j \neq i} \rho(\mathbf{r}_{ij}) \right)$$

Integration of Newton's equations: Euler vs Verlet

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Euler (order 1)

$$\mathbf{x}(t + \Delta t) =$$

$$\mathbf{v}(t + \Delta t) =$$

Verlet (order 2)

$$\mathbf{x}(t + \Delta t) =$$

$$\mathbf{x}(t - \Delta t) =$$

$$\mathbf{x}(t + \Delta t) =$$

Modified Euler (order ?)

$$\mathbf{x}(t + \Delta t) =$$

$$\mathbf{v}(t + \Delta t) =$$

$$\mathbf{x}(t + \Delta t) =$$

Molecular dynamics

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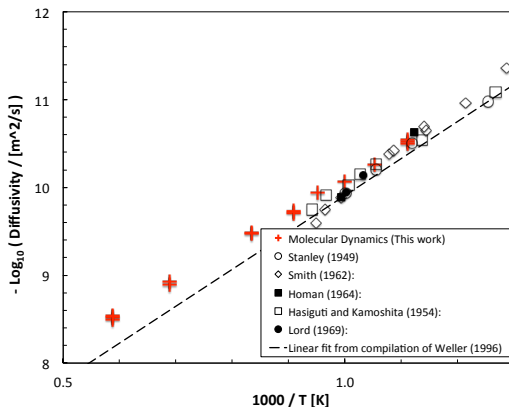
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Results: let the carbon move!

Diffusivity (from Einstein's relation): $6DT = \langle |r(t) - r(0)|^2 \rangle$



Molecular dynamics

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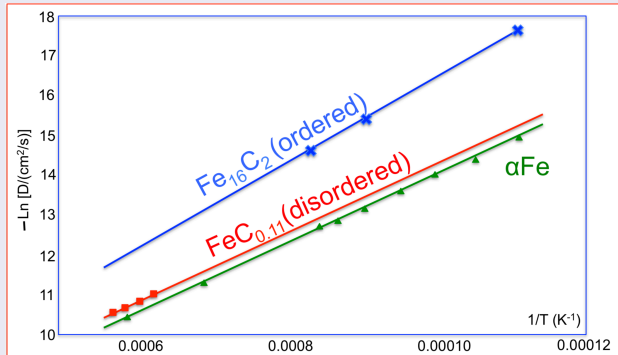
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Results: let the carbon move!

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Diffusivity of C in various phases

Atomic Kinetic Monte-Carlo

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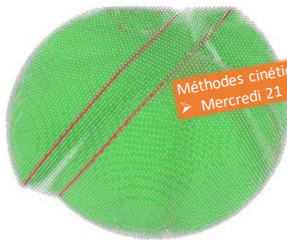
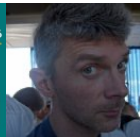
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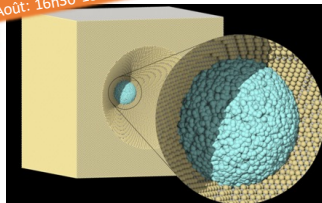
Conclusions



Laurent Pizzafalli
Institut P' (Poitiers)

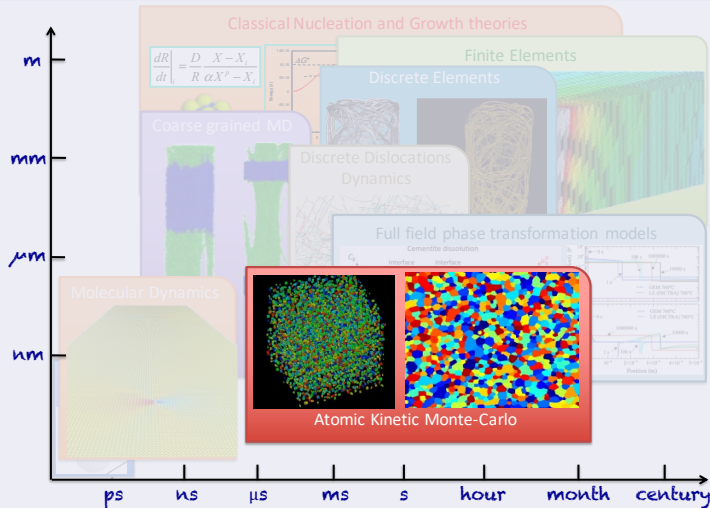


Méthodes cinétiques: dynamique moléculaire
➤ Mercredi 21 Août: 16h30-18h30



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Monte-Carlo Metropolis

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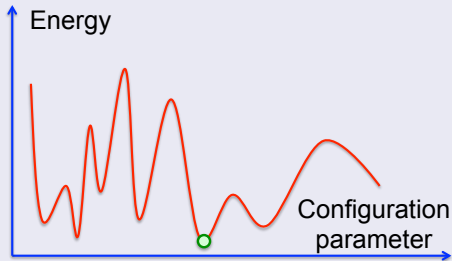
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Conclusions

What is the most stable phase?



- Energy:

$$E(x, y, z, v_x, v_y, v_z)$$

- Values of x, y, z, v_x, v_y and v_z that minimize E ?

Monte-Carlo Metropolis

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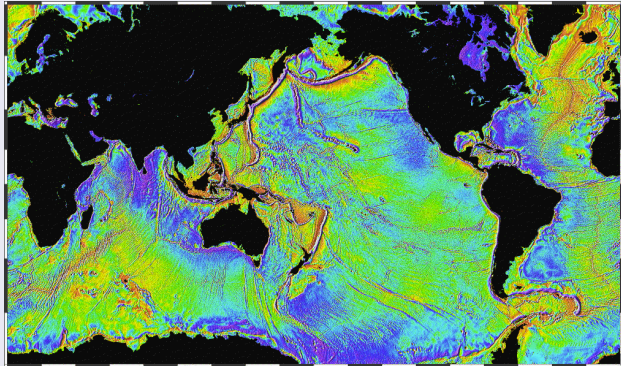
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Example: what is the deepest point in the ocean?



Monte-Carlo Metropolis

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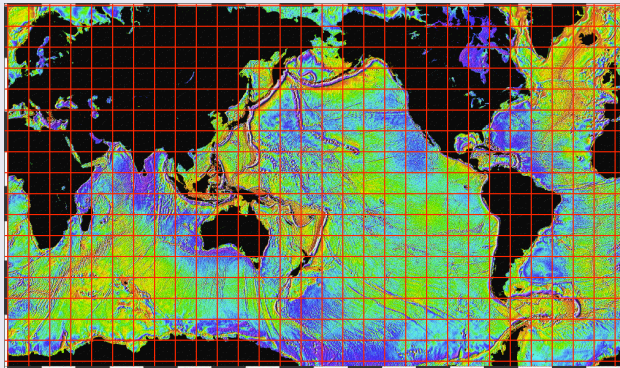
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Example: what is the deepest point in the ocean?



😊 Sure we will find it!

😞 At what cost?

Monte-Carlo Metropolis

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Multi-scale

Ab-initio

MD

Monte-Carlo

Full Field

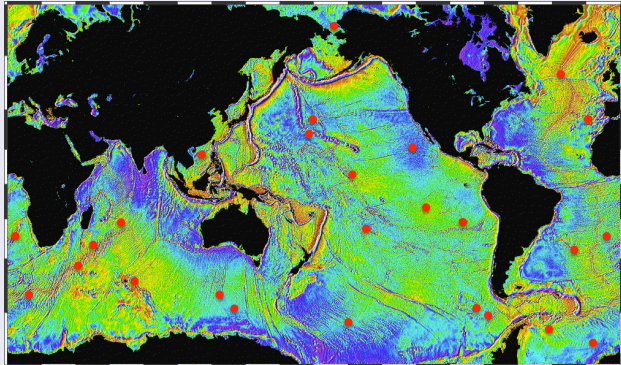
CNGTs

DDD

FEM

Conclusions

Example: what is the deepest point in the ocean?



😊 Low probability

😞 Cost is controlled

$$P_{i \rightarrow j} = \min(1, \exp[-\Delta E_{ij}/kT])$$

Monte-Carlo Metropolis

Multi-scale
approaches

Modeling

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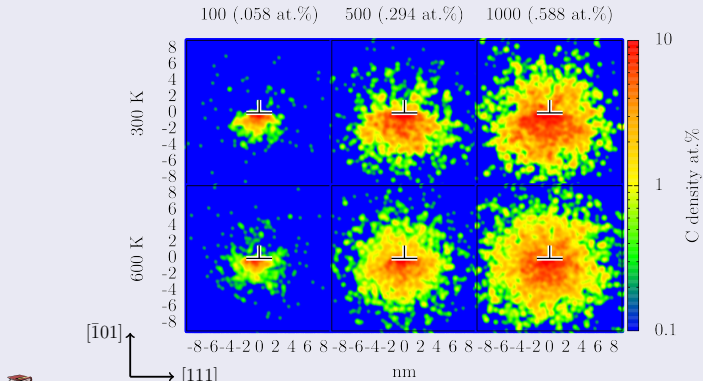
CNGTs

DDD

FEM

Conclusions

Application: build up of a Cottrell atmosphere



[Waseda et al, *Scripta Mater.* **129** (2016)]

- ≈ 150 C atoms per nm (same as measured with TAP)
- Max. C concentration: 8.8% (TAP: 8%)

Atomic Kinetic Monte-Carlo

Multi-scale
approaches

Modeling

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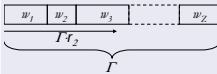
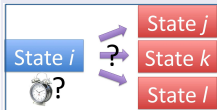
CNGTs

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Conclusions

Principle

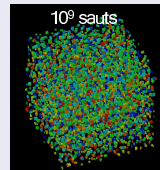
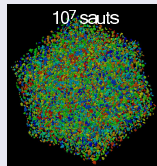
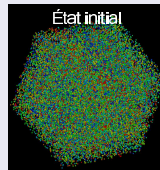


- Transition frequency from $i \rightarrow j$:

$$w_j^i = w_0 \exp\left(\frac{-\Delta E_{ij}}{kT}\right) \quad w_0 \approx 10^{14} \text{ Hz}$$
- Residence time i

$$r_1 \rightarrow \tau_R = -\frac{\ln r_1}{\sum_j w_j}$$
- Choice of a particular transition r_2

Example: precipitation from a supersaturated solid solution



Atomic Kinetic Monte-Carlo

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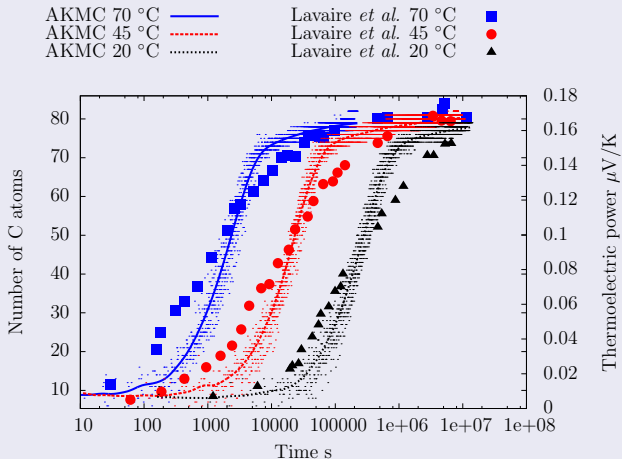
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Conclusions

Formation kinetics: AKMC vs ThermoElectric Power



- Same kinetics **without any adjustable parameter**

Atomic Kinetic Monte-Carlo

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Conclusions



**Anne HEMERYCK et
Antoine JAY**
LAAS-CNRS, Toulouse



Méthodes cinétiques : Transition State Theory,
Monte Carlo cinétique, paysage énergétique,
exploration avec la méthode ARTn

➤ mercredi 21 Août: 8h-10h

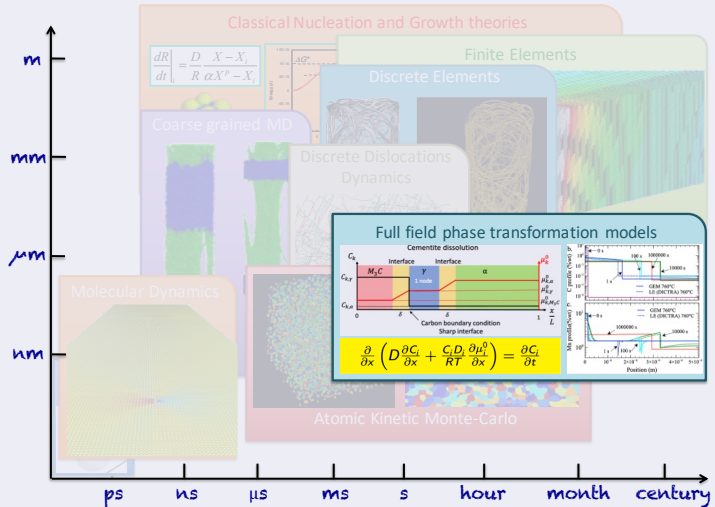
Exploration du Paysage Énergétique avec ARTn:
recherche des point selles
Monte Carlo cinétique : évolution des
structures avec le temps

➤ Lundi 19 Août et Mardi 20 Août: 16h-19h



Multi-scale approach

Different scales in material science



Multi-scale
approaches

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Full Field models ($\alpha \leftrightarrow \gamma$ transformation)

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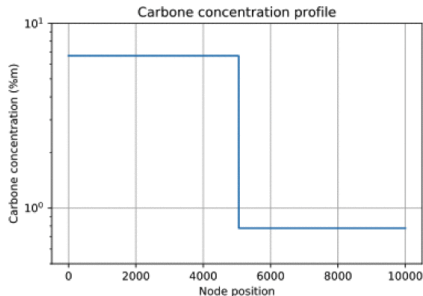
FEM

Conclusions

First ingredient: diffusion of solute elements (Fick's law)

- Classical diffusion model (implicit resolution scheme):

$$\frac{\partial C_i}{\partial t} = D_i \frac{\partial^2 C_i}{\partial x^2} \Rightarrow \frac{C_i^{t+\Delta t} - C_i^t}{\Delta t} = D_i \frac{C_{i+1}^{t+\Delta t} - 2C_{i+1}^{t+\Delta t} + C_{i-1}^{t+\Delta t}}{\Delta x^2}$$



Full Field models ($\alpha \leftrightarrow \gamma$ transformation)

Multi-scale
approaches

Second ingredient: an interface

- Interface = chemical potential variation

$$\frac{\partial C_i}{\partial t} = \frac{\partial}{\partial x} \left(D \frac{\partial C_i}{\partial x} \right)$$

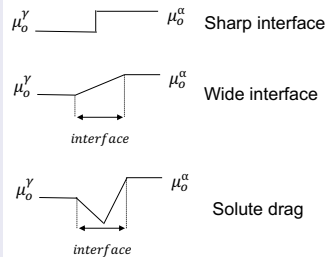


$$\frac{\partial C_i}{\partial t} = \frac{\partial}{\partial x} \left(\frac{D_i C_i}{RT} \frac{\partial \mu_i}{\partial x} \right)$$

$$\mu_i = \mu_{0,i} + RT \ln(C_i)$$



$$\frac{\partial}{\partial x} \left(D \frac{\partial C_i}{\partial x} + \frac{C_i D_i}{RT} \frac{\partial \mu_i^0}{\partial x} \right) = \frac{\partial C_i}{\partial t}$$

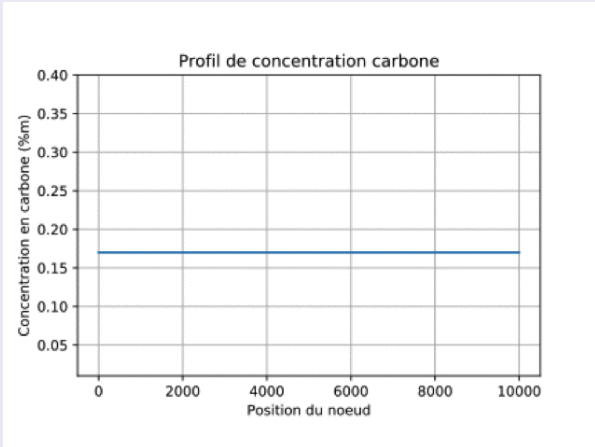


Full Field models ($\alpha \leftrightarrow \gamma$ transformation)

Multi-scale
approaches

Second ingredient: an interface !

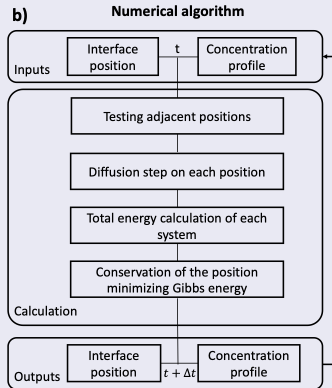
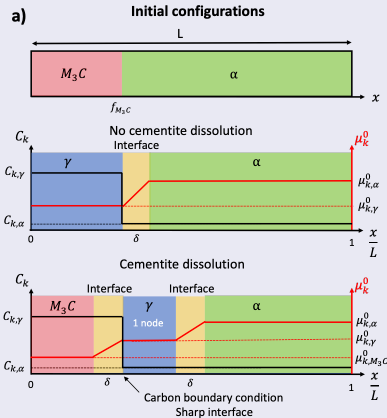
- Starting point: constant concentration



Full Field models ($\alpha \leftrightarrow \gamma$ transformation)

Multi-scale approaches

Third ingredient: moving the interface(s) !



- Choice of position that minimises the Gibbs energy
- Check that interface does not move too fast

Full Field models ($\alpha \leftrightarrow \gamma$ transformation)

Multi-scale
approaches

Results: austenite formation at 760°C (FeCMn steel)

Modeling

Aim

Who am I

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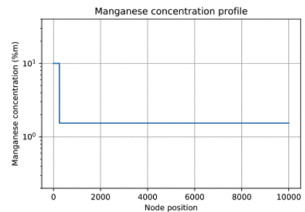
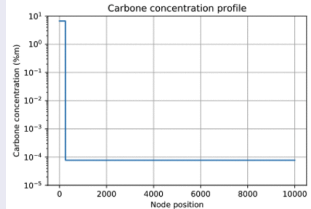
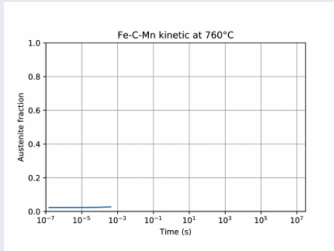
Full Field

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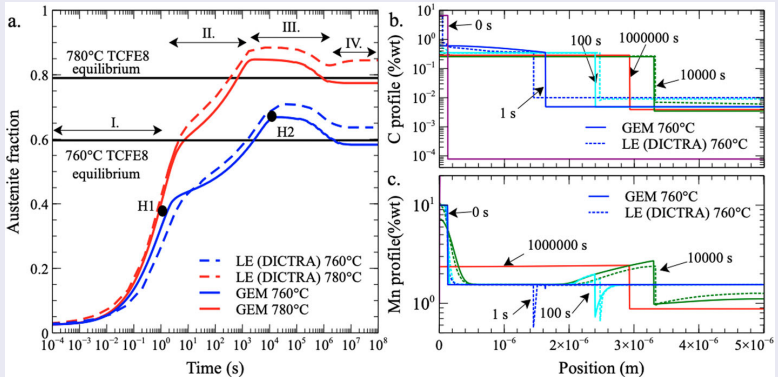
Conclusions



Full Field models ($\alpha \leftrightarrow \gamma$ transformation)

Multi-scale
approaches

Austenitic transformation kinetics (DICTRA vs GEM)



[A. Mathevons et al, Phil. Mag. Lett. **101** (2021)]

- Complex transformation kinetics due to 3 different diffusion coefficients

Full Field models

Multi-scale
approaches

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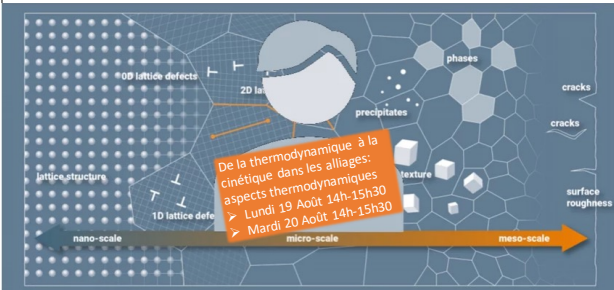


Mohamed Gouné

Institut de Chimie de la
matière Condensée de
Bordeaux

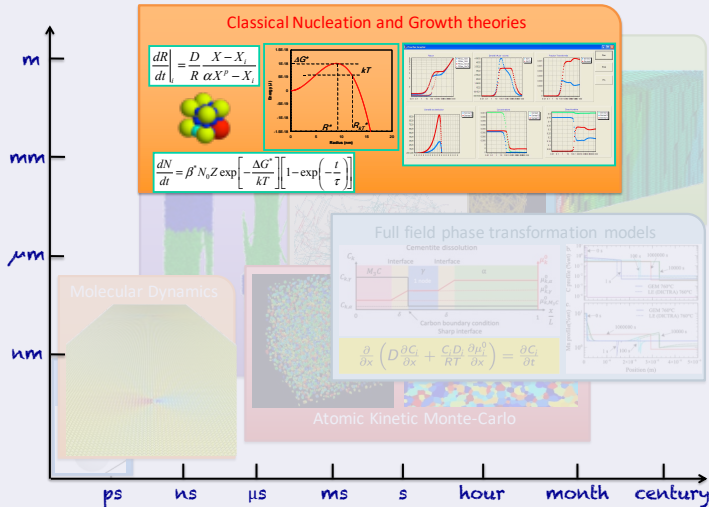


icmcb
institut de chimie de la matière condensée de bordeaux



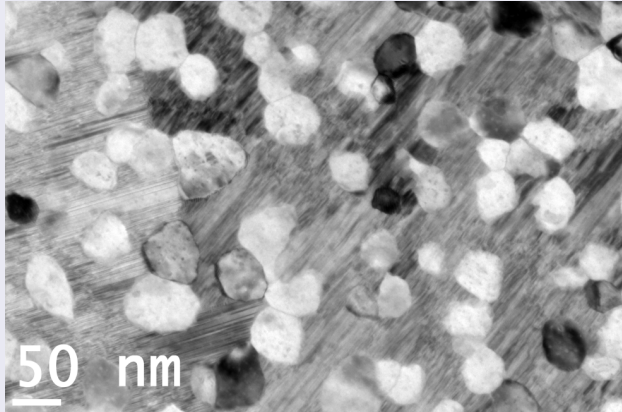
Multi-scale approach

Classical Nucleation and Growth Theories



Classical nucleation and growth theories

Mean field precipitation modelling



- What are these two phases?



Classical nucleation and growth theories

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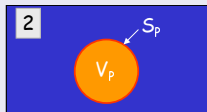
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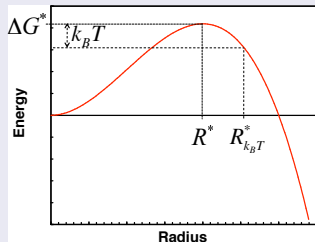
Conclusions

Nucleation rate



$$\Delta G = G_2 - G_1 = \frac{4}{3}\pi R^3 \Delta g + 4\pi R^2 \gamma$$

- Δg : driving force for nucleation (thermo. and/or Monte-Carlo)
- γ : interfacial energy



$$\frac{dN}{dt} = N_0 Z \beta^* \exp\left(-\frac{\Delta G^*}{kT}\right) \exp\left(-\frac{\tau}{t}\right)$$

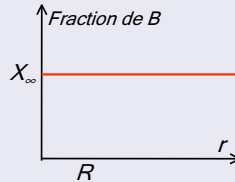
Classical nucleation and growth theories

Growth law: Linear out-of-equilibrium thermodynamics

Fick's law:

$$J = -D \frac{\partial C}{\partial r}$$

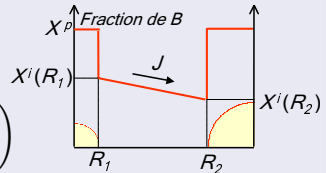
$$\frac{dR}{dt} = \frac{D}{R} \frac{X - X^i(R)}{X^p - X^i(R)}$$



Effet of precipitate size (Gibbs-Thomson)

- Precipitate composition $A_x B_y$

$$X^i(R) = X^i(\infty) \exp \left(\frac{(x+y)R_0}{yR} \right)$$



[M. Perez, Scripta Mat. **52** (2005)]

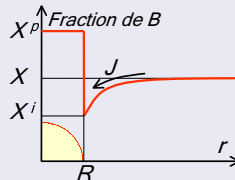
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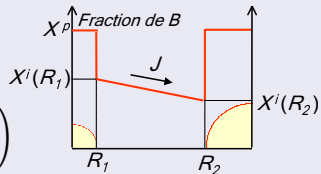
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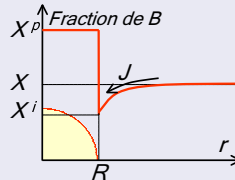
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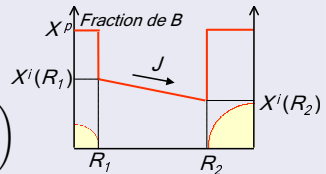
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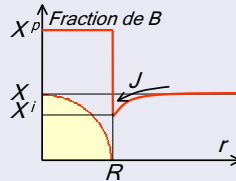
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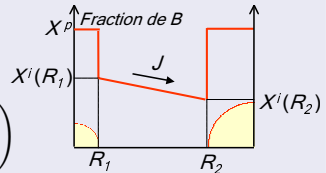
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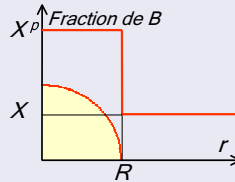
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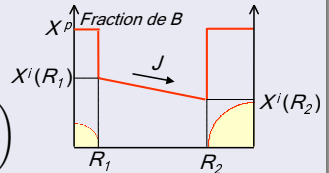
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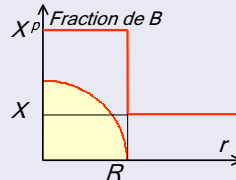
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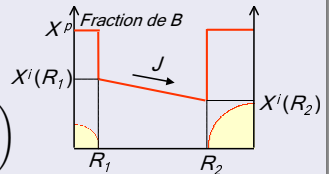
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[M. Perez, Scripta Mat. **52** (2005)]

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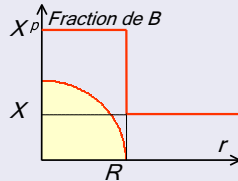
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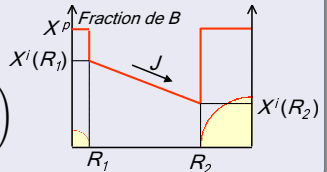
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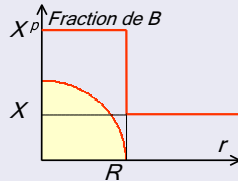
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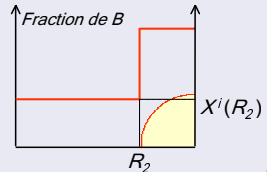
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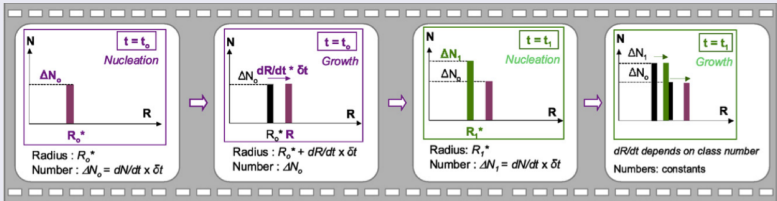
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Classical nucleation and growth theories

Multi-scale
approaches

Implementation in a home made precipitation software: PreciSo



Whole precipitate size distribution evolution

for non-isothermal treatments



[M. Perez et al, Acta Mater. **56** (2008)]

Classical nucleation and growth theories

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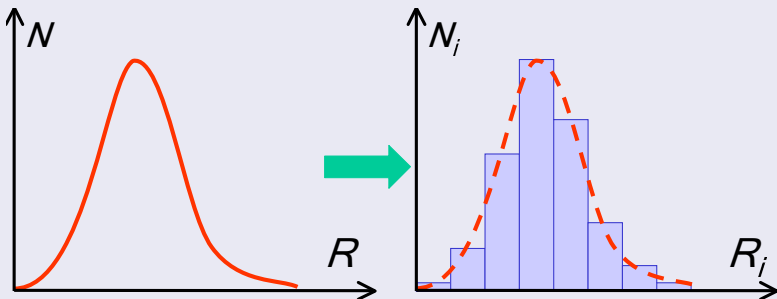
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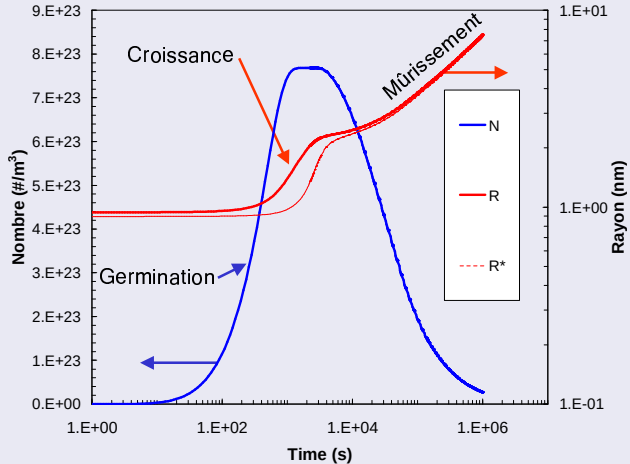


[M. Perez et al, Acta Mater. **56** (2008)]

Classical nucleation and growth theories

Multi-scale
approaches

Example of precipitation kinetics



CNGTs vs Full Field

Multi-scale
approaches

Modeling

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Full Field

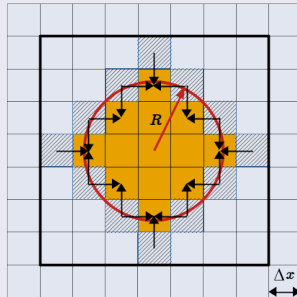
CNGTs

DDD

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Conclusions

Sharp Interface Full field mModel (ShIFuMo)



- Nucleation: Classical Nucleation Theory
- Growth: Classical Growth Theory (Zener)
- But: 3D diffusion: Full Field

CNGTs vs Full Field

Multi-scale
approaches

Modeling

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Full Field

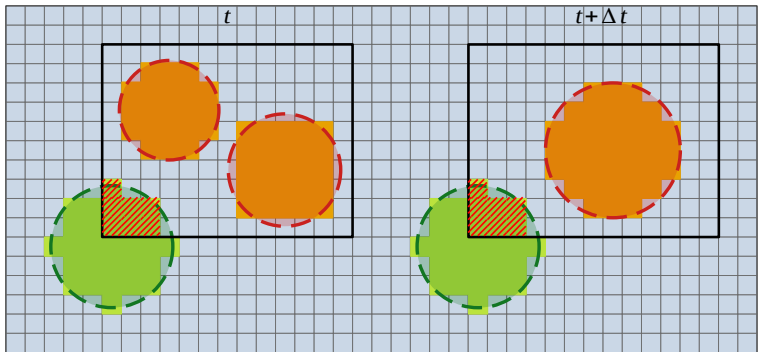
CNGTs

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Coagulation



CNGTs vs Full Field

Comparison

Multi-scale
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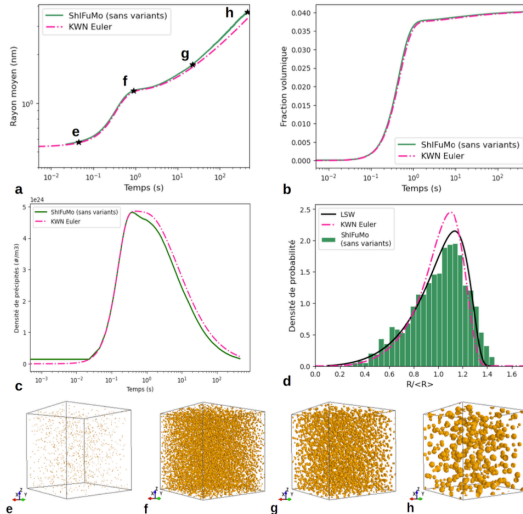
Full Field

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CNGTs vs Full Field

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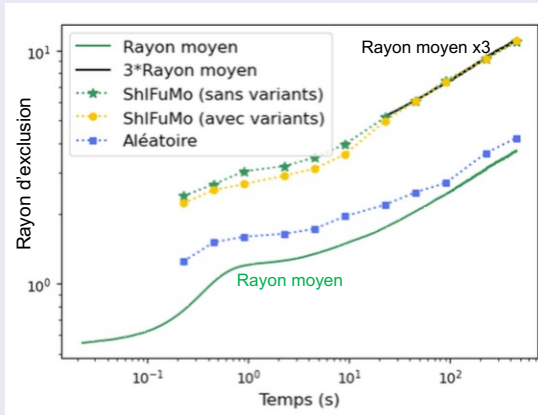
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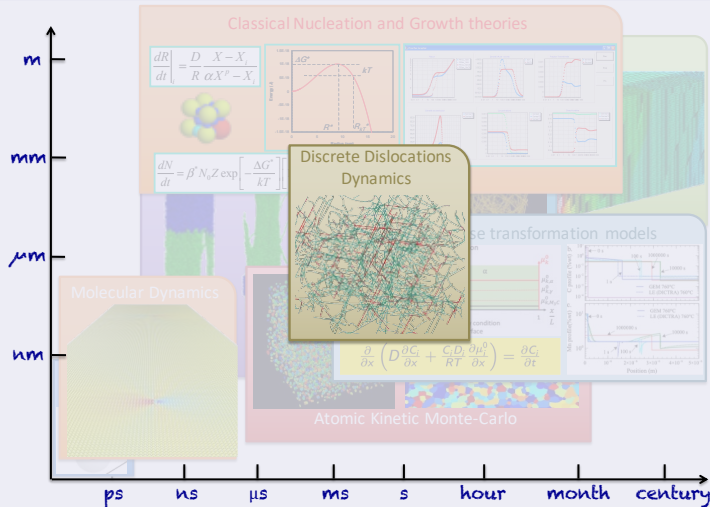
Coagulation



- Improve CNGTs from Sharp Interface Full Field model !

Multi-scale approach

Discrete Dislocations Dynamics



Multi-scale
approaches

Modeling

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Discrete Dislocations Dynamics

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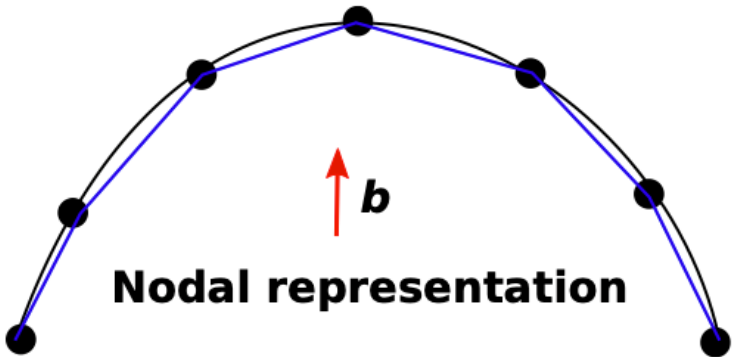
CNGTs

DDD

FEM

Conclusions

Presentation



- Dislocation: discretised into segments (mixed dislocation)

Discrete Dislocations Dynamics

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Conclusions

Effective net force per unit length

$$\mathbf{F}_{\text{eff}} = \underbrace{\sigma \mathbf{b} \times \boldsymbol{\xi}}_{\text{Peach-Koehler}} + \underbrace{F_{\text{lt}}}_{\text{Line tension}} - \underbrace{\frac{\mathbf{v}}{|\mathbf{v}|} F_{\text{peierls}}}_{\text{Peierls friction}}$$

Dynamics

$$\mathbf{v} = \frac{\mathbf{F}_{\text{eff}}}{B}$$

Stress and strain update

$$\sigma = \sum_{\text{nodes}} \delta \sigma_i \text{ and } \epsilon = \sum_{\text{nodes}} \delta \epsilon_i$$

Discrete Dislocations Dynamics

Exemple of Numodis (CEA) algorithm

Numodis initialization

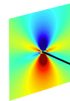
Discretization

Stress calculation

Force calculation

Velocity calculation

Discrete events



$$\mathbf{f} = (\boldsymbol{\sigma} \cdot \mathbf{b}) \times \boldsymbol{\xi}$$

$$\mathbf{v} = \mathbf{M}(\mathbf{f})$$



Multi-scale
approaches

Modeling

Aim

Who am I

Multi-scale

Ab-initio

MD

Monte-Carlo

Full Field

CNGTs

DDD

FEM

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Discrete Dislocations Dynamics

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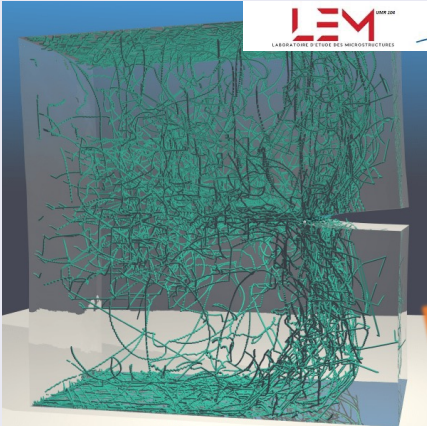
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LEM UMR 2004
LABORATOIRE D'ETUDE DES MICROSTRUCTURES

ONERA
THE FRENCH AEROSPACE LAB

cnrs



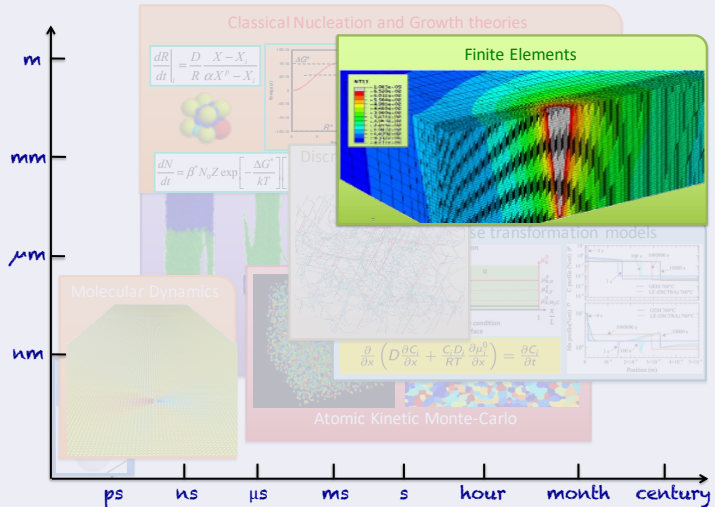
Riccardo Gatti

Méthodes continues
(éléments finis, DDD),
plasticité

➤ Jeudi 22 10h30-
12h30

Multi-scale approach

Different scales in material science



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Newton's second law

$$\rho \frac{\partial^2 u}{\partial t^2} - \nabla \sigma = \mathbf{f}$$

Constitutive law (e.g. thermo-elastic)

$$\sigma^{ij} = C^{ijkl} \epsilon^{kl} - \beta^{ij} (T - T_0)$$

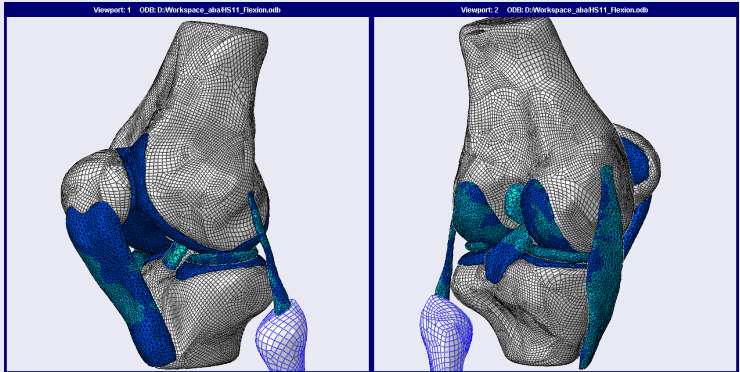
Linearized strain

$$\epsilon = \frac{1}{2} (\nabla u + \nabla u^T)$$

Finite Elements Method

Multi-scale
approaches

Mesh of a meniscus



[From <https://ris.utwente.nl/ws/files/6153316/CMBBE2014-Hamid-Submitted.pdf>]

Finite Elements Method

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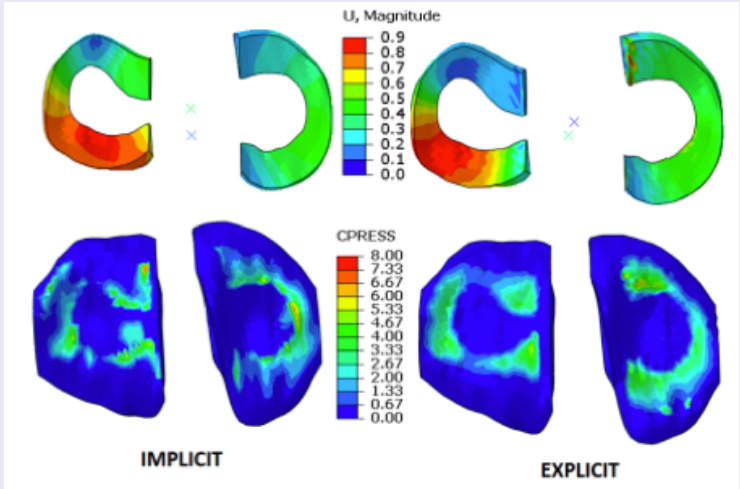
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Conclusions

Displacements and pressure on the meniscus



Finite Elements Method and DDD coupling

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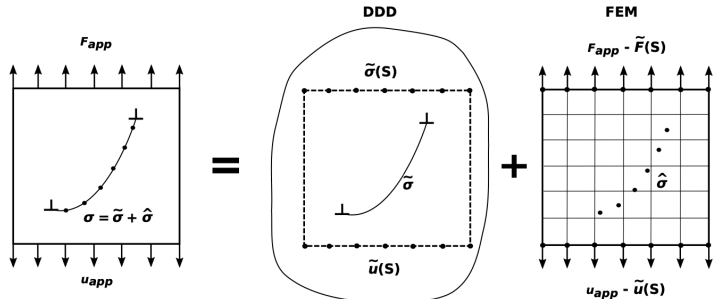
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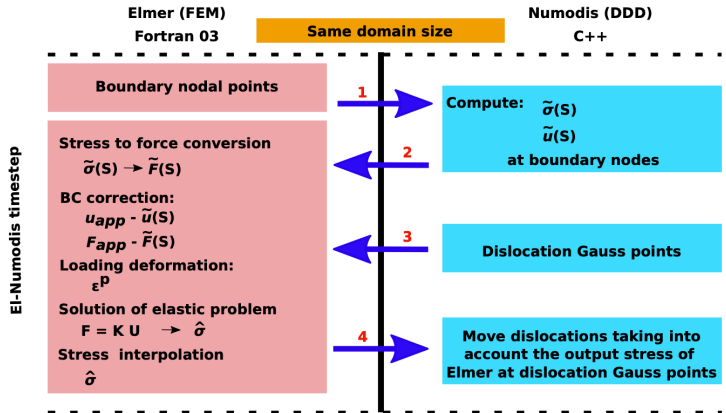
Two complementary methods



Finite Elements Method and DDD coupling

Multi-scale
approaches

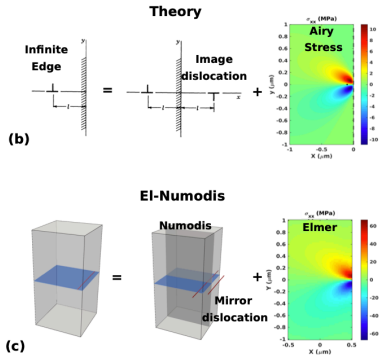
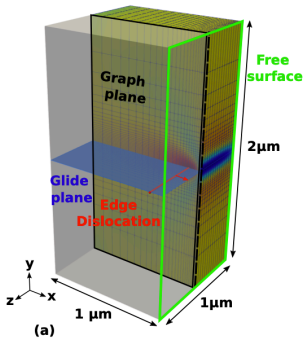
Principle



Finite Elements Method and DDD coupling

Multi-scale
approaches

Validation



Finite Elements Method and DDD coupling

Multi-scale
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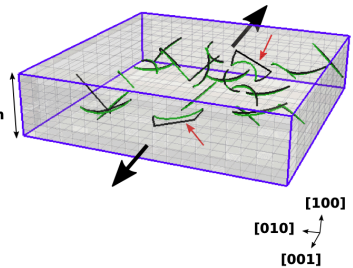
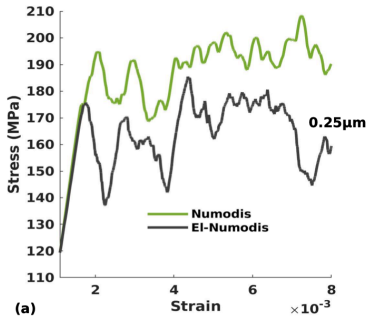
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Results



The icing on the cake...

Multi-scale approaches

Modeling

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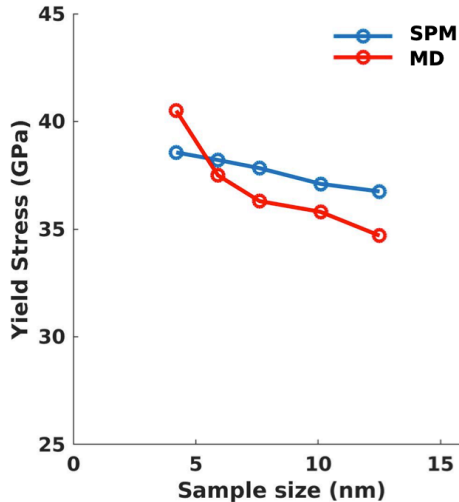
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Comparison with MD



Last break !

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Conclusions



Lucia Sargentini

Univ. Paris Saclay CEA



Statistical methods for the quantification of uncertainty in materials
Méthodes statistiques pour la quantification/propagation d'incertitudes dans les matériaux
Lucia Sargentini
University of Paris Saclay CEA
Service de Mécanique des Matériaux
91191, Gif-sur-Yvette
Vendredi 22 Août: 8h-10h

1. Calibration under uncertainty

- The importance of the experimental data
- The importance of the physics
- The importance of the statistical model

2. Uncertainty of assessed model

- The importance of sensitivity analysis
- The importance of the statistical model for the quantification of uncertainty

Conclusions

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Make your own conclusion



Thank you!

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The METAL team



{Team building 2023}