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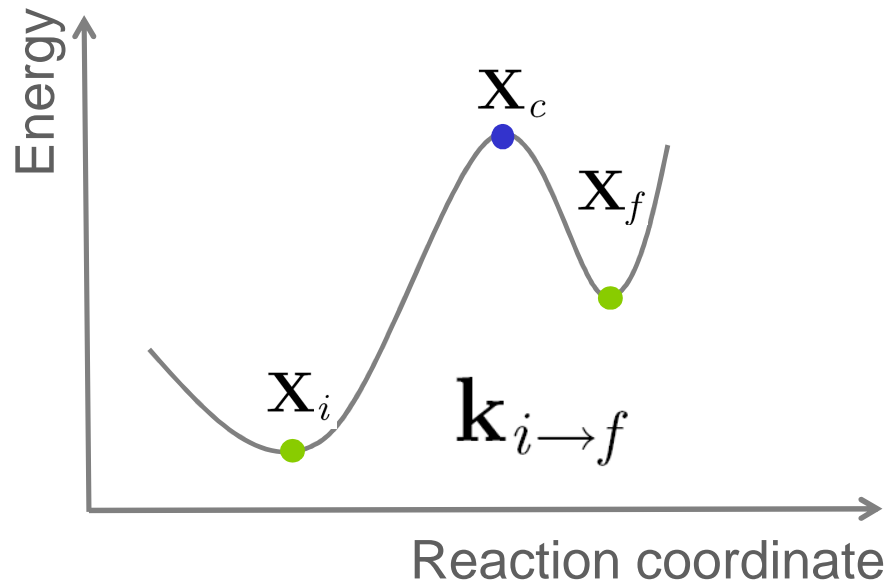
# ***EXPLORING THE ENERGY LANDSCAPE***

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Saclay, France**

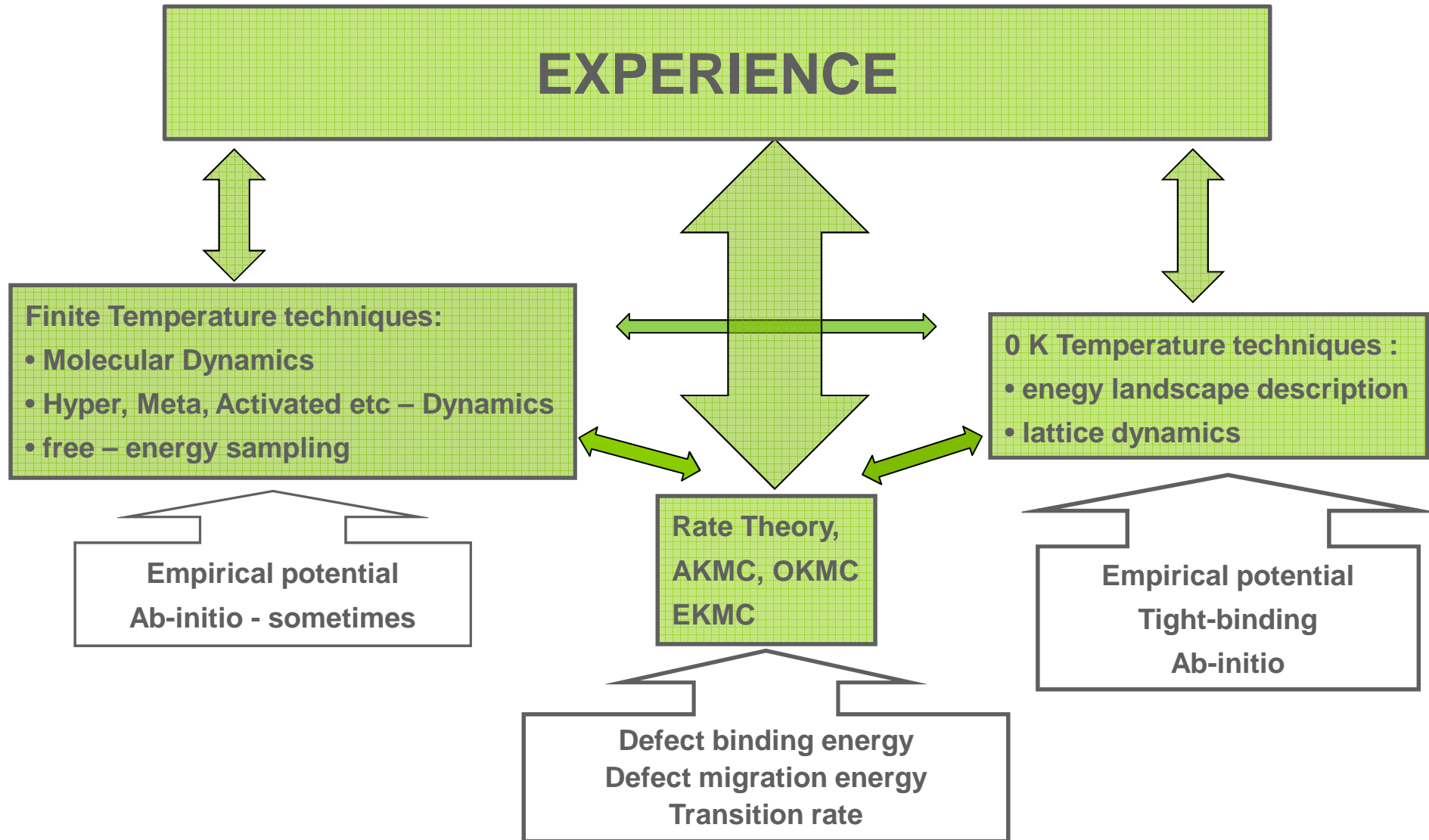
[mihai-cosmin.marinica@cea.fr](mailto:mihai-cosmin.marinica@cea.fr)

The goal is to have the values for transition rates



... which can be used in any large scale is time and space simulation:

1. Rate theory
2. AKMC,
3. OKMC,
4. EKMC (JERK, FPKMC)
5. ...



$$H(q, p) = \frac{1}{2} p^T M^{-1} p + V(q) \quad M = \text{Diag}(m_1, \dots, m_N) \quad q = (q_1, \dots, q_N) \in \mathbb{R}^{3N}$$

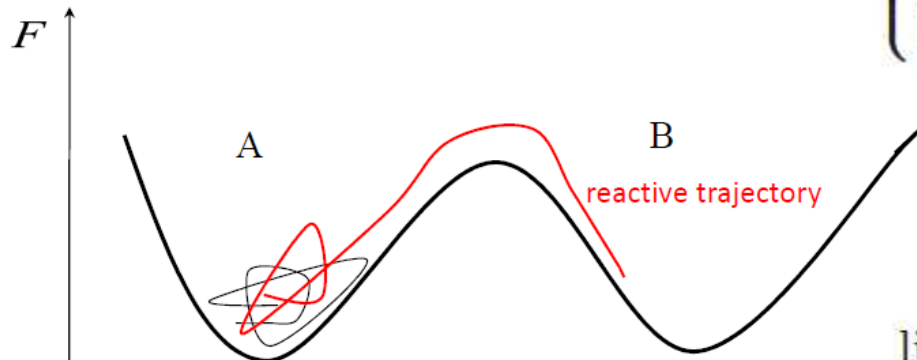
$$(p_1, \dots, p_N) \in \mathbb{R}^{3N}$$

$$\langle A \rangle = \int_{\mathcal{M}^N \times \mathbb{R}^{3N}} A(q, p) d\mu(q, p).$$

$$d\mu(q, p) = Z^{-1} \exp(-\beta H(q, p)) dq dp,$$

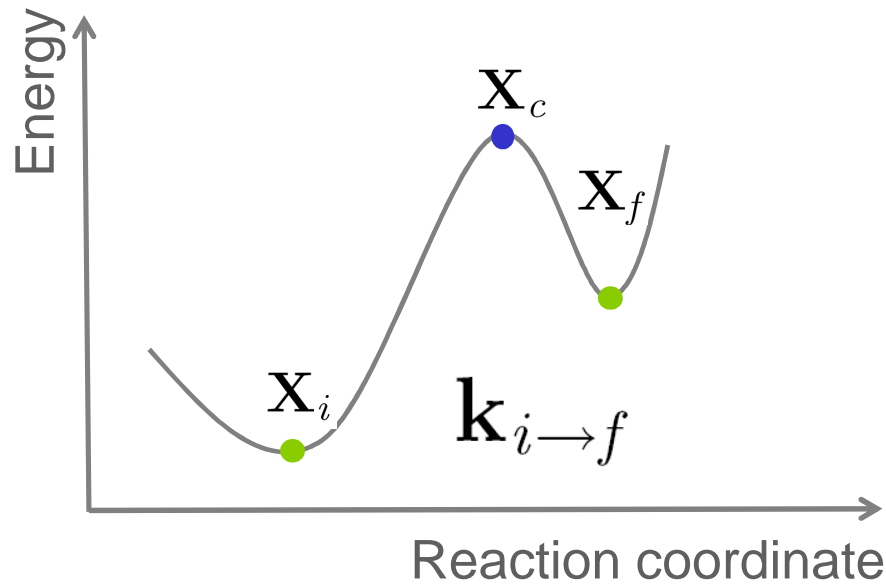
$$Z = \int_{T^* \mathcal{M}} \exp(-\beta H(q, p)) dq dp.$$

$$\begin{cases} \dot{q}_i(t) = \frac{\partial H}{\partial p_i}(q(t), p(t)) = \frac{p_i(t)}{m_i}, \\ \dot{p}_i(t) = -\frac{\partial H}{\partial q_i}(q(t), p(t)) = -\nabla_{q_i} V(q(t)). \end{cases}$$



$$\lim_{N \rightarrow +\infty} \frac{1}{N} \sum_{n=0}^{N-1} A(q^n, p^n) = \int_{T^* \mathcal{M}} A(q, p) d\mu(q, p)$$

The goal is to have the values for transition rates



... which can be used in any large scale is time and space simulation:

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5. ...

$$\begin{aligned}
 k_{i \rightarrow f} &= k \times e^{-\frac{F_c - F_i}{k_B T}} \\
 &= k e^{\frac{S_c - S_i}{k_B}} \times e^{-\frac{E_c - E_i}{k_B T}} \\
 &= \nu_0 \times e^{-\frac{E_c - E_i}{k_B T}}
 \end{aligned}$$

**Le but:** trouver sur une surface d'énergie potentielle (« PES »):

➤ Les minima  $\mathbf{X}$  :  $f(\mathbf{X}) = 0$  et  $\lambda_1 > 0$

$\mathbf{X} = (q_1, q_2, \dots, q_N)$  - la configuration

$\mathbf{f}(\mathbf{X}) = -\nabla E(\mathbf{X})$  - le gradient

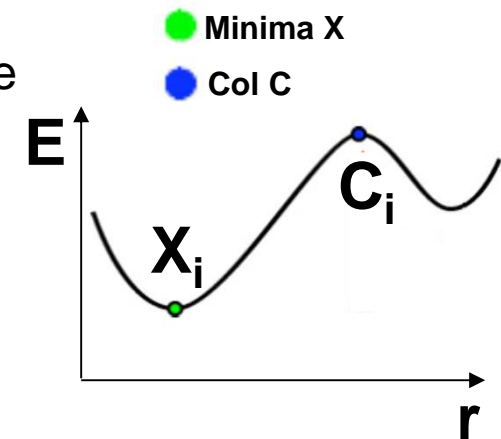
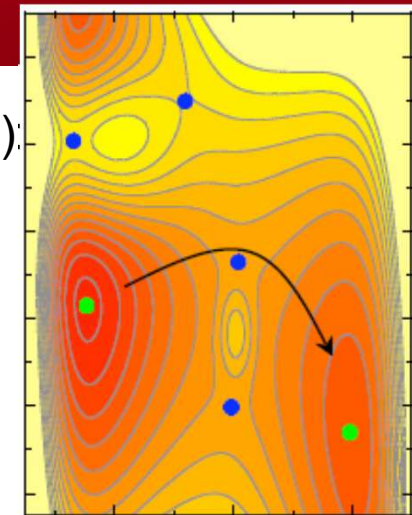
$$\mathbf{H}(\mathbf{X}) = \left\| \frac{\partial^2 E}{\partial q_{i\alpha} \partial q_{j\beta}} \right\|$$

$$\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_{3N}$$

- le Hessien et son spectre

➤ Les points de cols  $\mathbf{C}$ :  $f(\mathbf{C}) = 0$  et  $\lambda_1 < 0$

Dans la suite nous ne faisons pas la différence entre les points de col de 1er, 2ème ... ordre



➤  $\mathbf{X}_I$  et  $\mathbf{X}_F$  sont connues :

- Application directe du principe de Hamilton: minimisation de l'action de la trajectoire qui lie les deux bassins

$$\delta S = 0$$

$$S = \int_0^t L(\mathbf{X}(\tau), \dot{\mathbf{X}}(\tau)) d\tau \approx \frac{t}{N+1} \sum_{J=0}^N L_J(\mathbf{X}_J, \dot{\mathbf{X}}_J)$$

$$\mathbf{X}(0) = \mathbf{X}_I$$

$$\mathbf{X}(t) = \mathbf{X}_F$$

- Exemples:

- relaxation sous contraintes: Bennet, drag

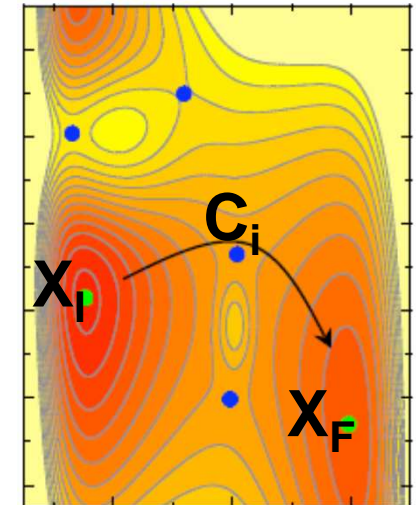
$$S = \sum_J E(\mathbf{X}_J) \quad (\mathbf{X}_J - \mathbf{X}_{CM}) (\mathbf{X}^F - \mathbf{X}^I) = 0$$

- les méthodes de chaînes (Ulitsky-Elber, NEB, CI-NEB etc)

$$S = \sum_J E(\mathbf{X}_J) + \frac{1}{2}k \sum_J (\|\mathbf{X}_{J+1} - \mathbf{X}_J\|^2 - \|\mathbf{X}_J - \mathbf{X}_{J-1}\|^2)$$

- Dynamique moléculaire par Action (ADM)

$$S = \frac{t}{N+1} \sum_{J=0}^N L_J(\mathbf{X}_J, \dot{\mathbf{X}}_J) \quad L_J(\mathbf{X}_J, \dot{\mathbf{X}}_J) \approx L_J(\mathbf{X}_J, \mathbf{X}_{J\pm 1})$$



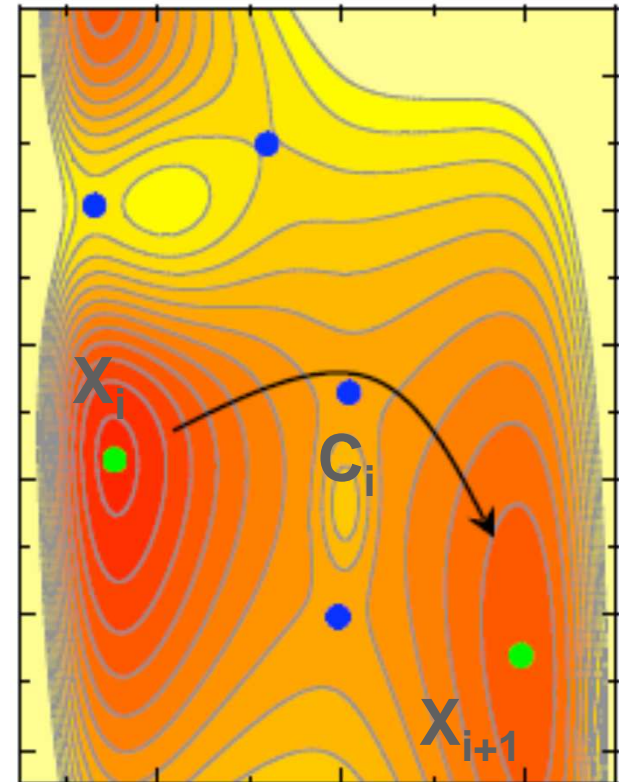
● Minima X

● Col C

# The Activation Relaxation Technique (ART)

A method to explore a potential energy surface: search for saddle points and local minima

A. Barkema, N. Mousseau PRL (1998) ; Phys. Rev. B (2000)  
E. Cancès, F. Legoll, M.C. Marinica, F. Willaime, J. Chem. Phys (2009)  
M.C. Marinica, F. Willaime, N. Mousseau, Phys. Rev. B, (2011)



- **$X_i$  is known** the others minima  $\{X_i\}$  and the saddle points  $\{C_i\}$  must be revealed by the method:
  - 0 K method, local information; partial Hessian
  - Fast, adapted for defects
  - Tested for Fe, Cu, Zr, W, Si, SiC



Lanczos method: **ART**

▪ Hessian is required only in the j-dimension space  $j \ll 3N$  ( $j=15-30$ )

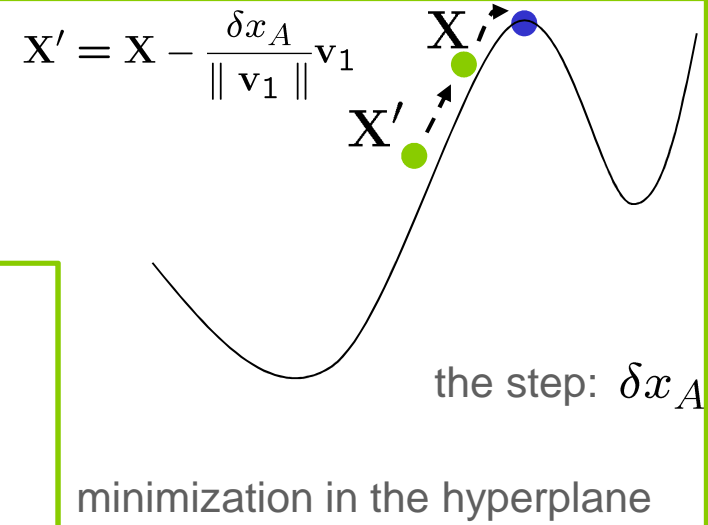
▪ diagonalization of the trigonal  $j \times j$  matrix

G. T. Barkema and N. Mousseau PRL (1998) ; PRB (2000)

Update 2015:

- **GAD** (W. E)
- **RAT** (R. Smith)
- **String M** (E. van Eijden)

The central point of the method: the evaluation of the lowest eigenvalue and the corresponding eigenvector  $\lambda_1$  and  $\mathbf{v}_1$

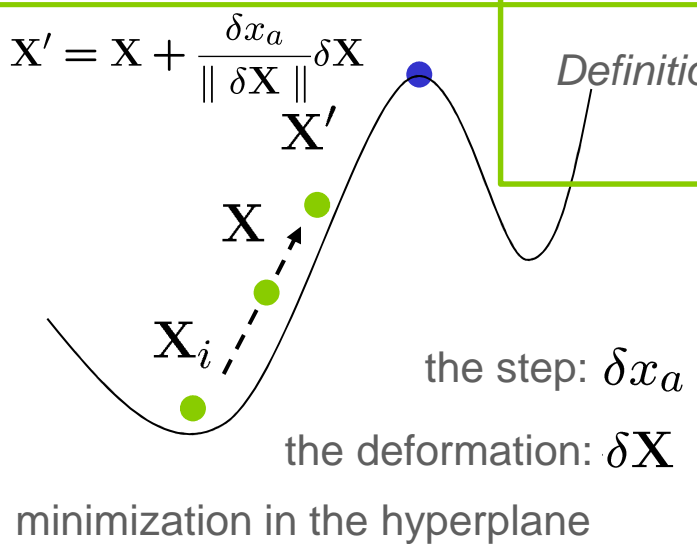


Rayleigh-Ritz **HEF**

▪ CG / L-BFGS optimization using the Rayleigh-Ritz criterion

D. Wales and J. Murno PRB (1998)

Definition of the inflexion point:  $\lambda_1 < -|\lambda_c|$



finite differences: **DIMER**

- the dimer is centered on  $\mathbf{X}$ :  $\mathbf{X} + \delta \mathbf{X}$  et  $\mathbf{X} - \delta \mathbf{X}$
- $\mathbf{v}_1$  (orientation of the dimer),  $\lambda_1$  (is estimated from the energy and the gradient)

Henkelman and Jonsson JCP, (1999)

$$\lambda_1 [\mathbf{T}_j] \rightarrow \lambda_1 [\mathbf{H}]$$

En général  $j=15-40$

$$\mathbf{T}_l = \begin{matrix} & (\mathbf{u}_0, \mathbf{u}_1, \dots, \mathbf{u}_{l-1}) \\ \begin{pmatrix} a_0 & b_1 & 0 & \dots & 0 \\ b_1 & a_1 & b_2 & \dots & 0 \\ 0 & b_2 & a_2 & \dots & 0 \\ 0 & \ddots & \ddots & \dots & 0 \\ 0 & & b_{l-2} & a_{l-2} & b_{l-1} \\ 0 & & 0 & b_{l-1} & a_{l-1} \end{pmatrix} \end{matrix}$$

$$\mathbf{H}\mathbf{u}_0 = a_0\mathbf{u}_0 + b_1\mathbf{u}_1$$

$$\mathbf{H}\mathbf{u}_1 = a_1\mathbf{u}_1 + b'_1\mathbf{u}_0 + b_2\mathbf{u}_2$$

$$\mathbf{u}_1 \cdot (\mathbf{H}\mathbf{u}_0) = \mathbf{u}_0 \cdot (\mathbf{H}\mathbf{u}_1)$$

...

$$\mathbf{H}\mathbf{u}_k = a_k\mathbf{u}_k + b_k\mathbf{u}_{k-1} + b_{k+1}\mathbf{u}_{k+1}$$

...

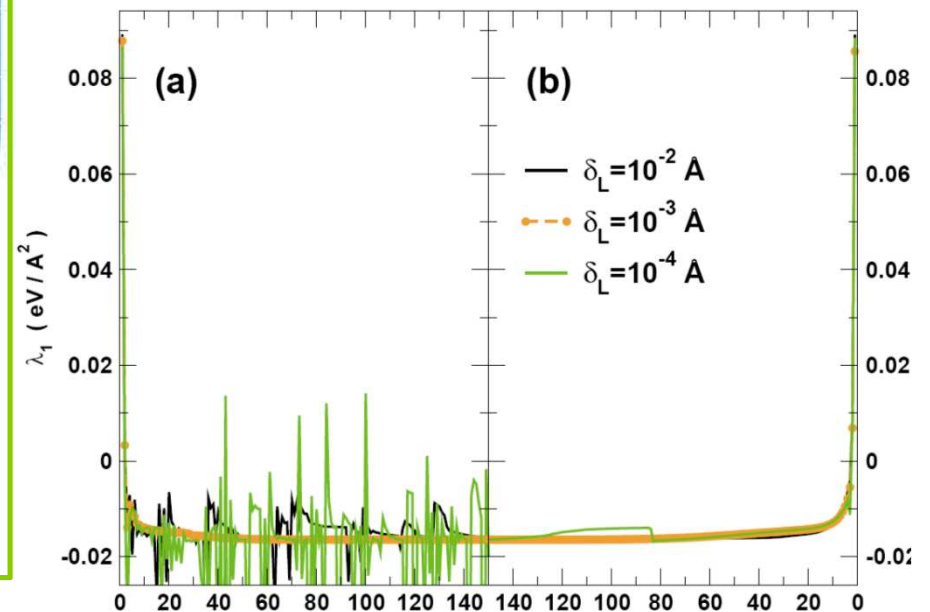
$$\mathbf{H}\mathbf{u}_{l-1} = a_{l-1}\mathbf{u}_{l-1} + b_{l-1}\mathbf{u}_{l-2}$$

we do not need entire H: only in the Krylov-Lanczos basis - dimension j

diagonalisation of trigonal  $j \times j$  matrix

$$H_{i\alpha, j\beta}[\mathbf{q}_0] = \frac{\partial^2 E[\mathbf{q}_0]}{\partial q_{i\alpha} \partial q_{j\beta}}$$

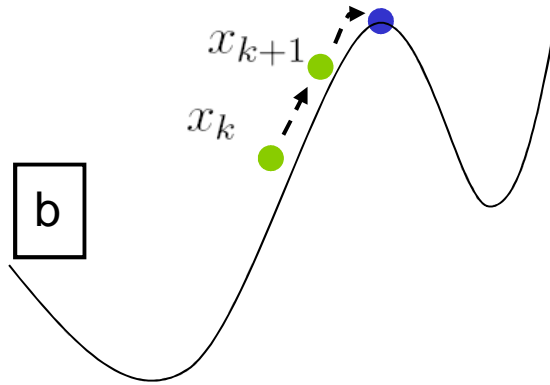
$$\mathbf{H}[\mathbf{q}_0] \mathbf{u} = - \frac{f(\mathbf{q}_0 + \delta_L \mathbf{u}) - f(\mathbf{q}_0)}{\delta_L}$$



$$\mathbf{H}[\mathbf{q}_0] \mathbf{u} = - \frac{f(\mathbf{q}_0 + \delta_L \mathbf{u}) - f(\mathbf{q}_0 - \delta_L \mathbf{u})}{2\delta_L}$$

1. The parameters which must be managed
  - The Lanczos step of the numerical derivative.
  - Size of the Lanczos basis

# The Activation Relaxation Technique (ART)



$$x_{k+1} = x_k - \frac{\mu_a}{\sqrt{k}} v_1(x_k) - \mu_t \Pi_{v_1(x_k)^\perp} \nabla f(x_k)$$

$$x_{k+1} = x_k - \frac{(\nabla f(x_k), v_1(x_k))}{\min(\lambda_1(x_k), -\lambda_c)} v_1(x_k) - \mu_t \Pi_{v_1(x_k)^\perp} \nabla f(x_k)$$

$$\Pi_{v_1(x_k)^\perp} = I - (v_1(x_k), \cdot) v_1(x_k)$$

$$x_{k+1} = x_k - \frac{\delta x_A}{\|v_1\|} v_1(x_k)$$

$$\delta x_A = \frac{\mu_a}{\sqrt{k}}$$

$$\delta x_A = \frac{(\nabla f(x_k), v_1(x_k))}{\min(\lambda_1(x_k), -\lambda_c)}$$

activation step  $\mu_a$

The numbers of distinct minima

0.05	1579	1
0.10	965	2
0.20	553	2
0.40	404	3
0.60	382	4
0.80	424	4
1.00	505	4

$\tilde{\lambda}_1(x_k)$  and  $\tilde{v}_1(x_k)$  are approximations of  $\lambda_1(x_k)$  and  $v_1(x_k)$

$$\tilde{v}_1(x_k) = v_1(x_k) + \alpha_k, \quad \tilde{\lambda}_1(x_k) = \frac{\lambda_1(x_k)}{1 + \beta_k}, \quad |\alpha_k| \ll 1 \text{ and } |\beta_k| \ll 1$$

$$|x_{k+1} - x_*| \leq \gamma |x_k - x_*| + O(|x_k - x_*|^2) + O(|x_k - x_*| |\alpha_k|) + O(|x_k - x_*| |\beta_k|),$$

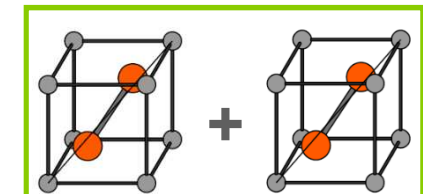
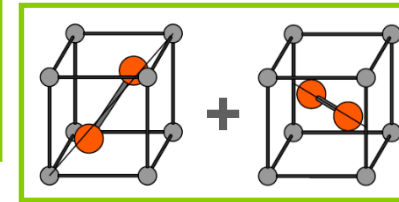
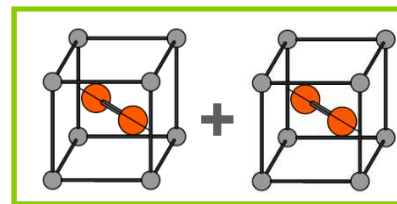
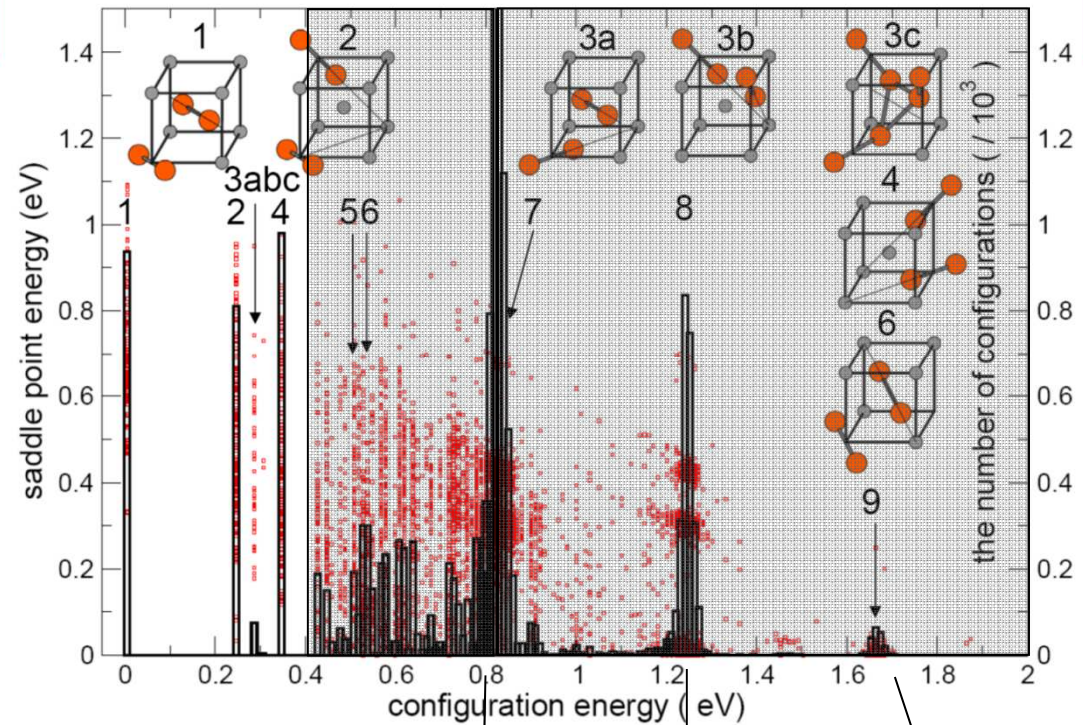
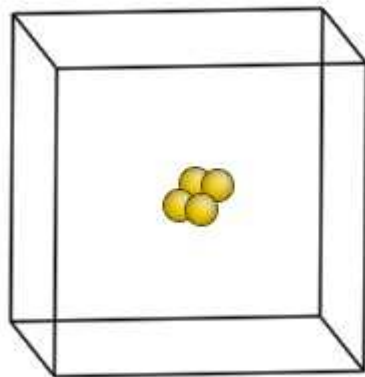
algorithm locally converges, and the convergence speed is at least linear.

number		<u>SIA</u>		<u>VAC</u>	
of defects		ART $n^{44}$	This work	ART $n^{44}$	This work
1	$\langle f \rangle$	462	298	780	291
	$\eta$	4.6	4.7	1.8	7.9
2	$\langle f \rangle$	548	328	705	323
	$\eta$	4.2	4.4	2.6	7.1
3	$\langle f \rangle$	691	320	667	321
	$\eta$	2.6	4.4	2.8	7.4

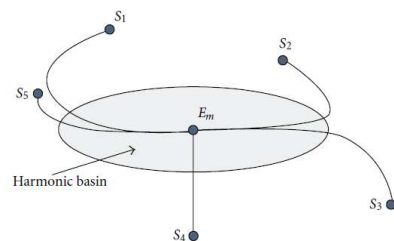
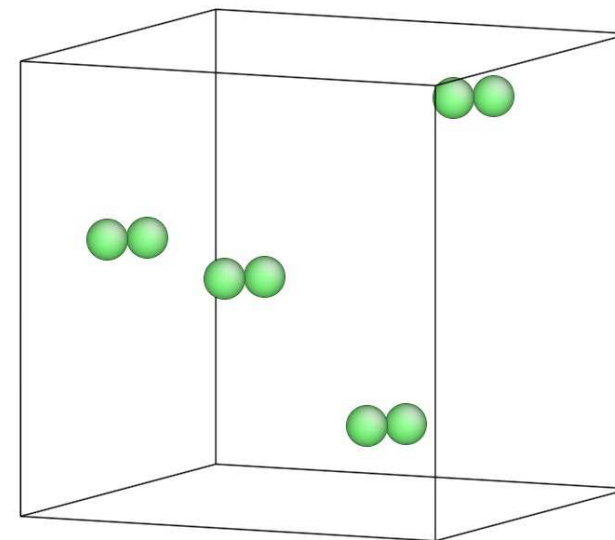
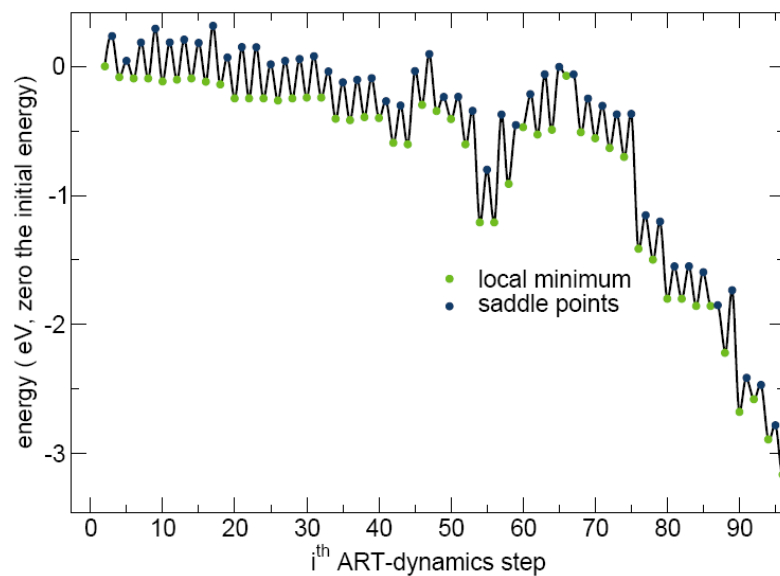
many non-parallel configurations between the  $I_2^{<110>}$  et  $I_2^{<111>}$

quasi-continuum of states between 0.42 eV and 0.83 eV (dissociation energy of the di-interstitiel)

beyond the bonding energy two separated SIA



Find the absolute minimum of an energy landscape



## Kinetic ART

N. Mousseau et al., *Journal of Atomic, Molecular, and Optical Physics*. 2012, 1–14 (2012).

1. **ART biblio:**

a. **ART :**

- ✓ G. T. Barkema, N. Mousseau, *Phys. Rev. Lett.* **81**, 1865 (1998).

b. **ART nouveau :**

- ✓ N. Mousseau, G. T. Barkema, *Phys. Rev. B* **61**, 1898-1906 (2000).
- ✓ A. Barkema, N. Mousseau *Phys. Rev. Lett.* (1998)
- ✓ E. Cances, F. Legoll, M.C. Marinica, F. Willaime, *J. Chem. Phys* (2009)
- ✓ M.C. Marinica, F. Willaime, N. Mousseau, *Phys. Rev. B*, (2011)

2. **Sources ARTn (C et F90, also interfaced with SIESTA, PWSCF)**

<http://www.phys.umontreal.ca/mousseau/index.php?n=Main.Logiciels>

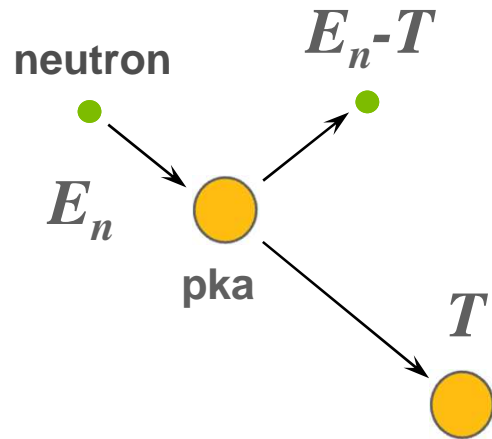
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**GOOGLE: ART Mousseau**

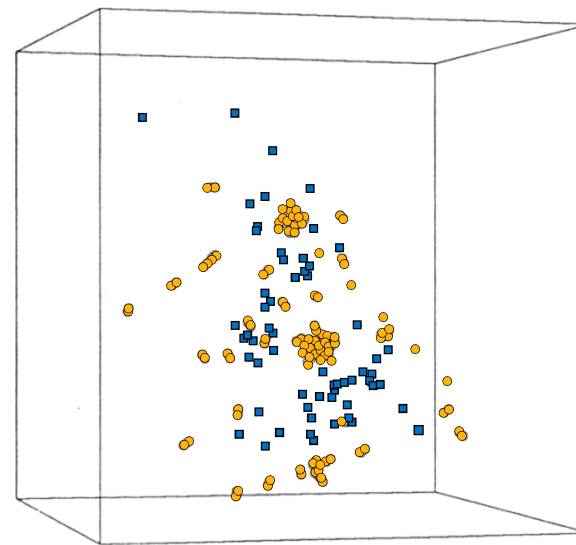
2. **kART, under request**

Materials under irradiation for the nuclear industry (e. g. high fluxes of neutrons)

Defect or Defect clusters



$T \gg E_d \Rightarrow$  displacement cascade



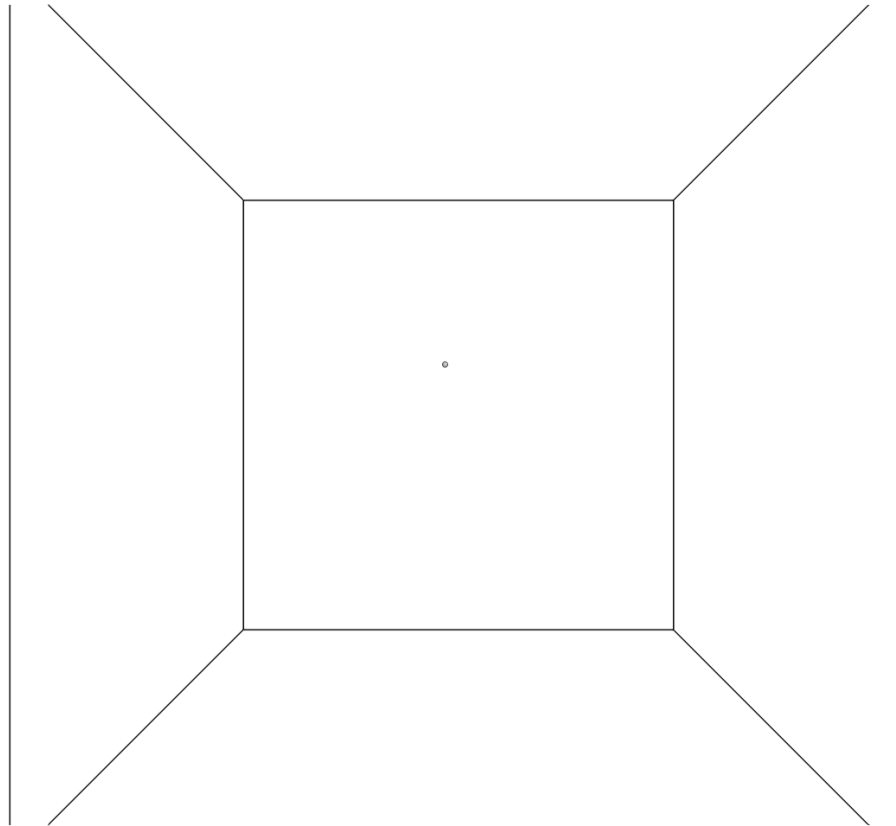
• interstitial  
▪ vacancy

- Isolated self-interstitials
- Isolated vacancies.
- Interstitial clusters.
- Vacancies clusters.
- +
- atomic mixing:  
(  $\approx 10$  Replacements / displacement )

pka = primary knocked atom  
T = kinetic energy transferred to the pka

Number of Frenkel pairs :  
 $n \approx \alpha T / (2.E_d)$

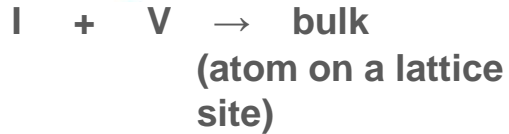
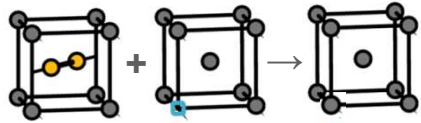




Molecular Dynamics simulations, 80 keV,  $3 \cdot 10^6$  atoms, 10 ps  
L. Van Brutzel (CEA/Saclay, France)

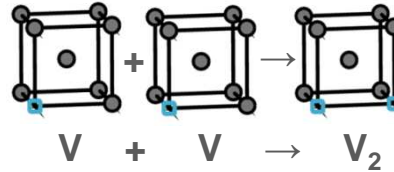
## □ Annihilation (recombination)

- system tend to recover the ground state (bulk state)

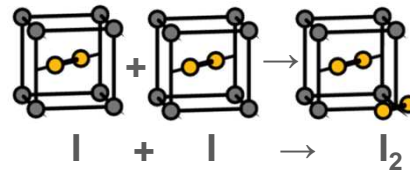


## □ Clustering (agglomeration)

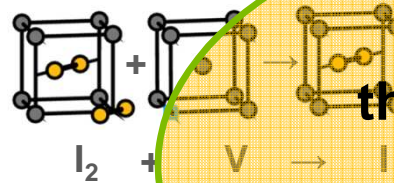
- vacancy :  $V_n + V_m \rightarrow V_{n+m}$



- interstitial :  $I_n + I_m \rightarrow I_{n+m}$



- annihilation:  $I_n + V_m \rightarrow I_{n-m} (V_{m-n})$

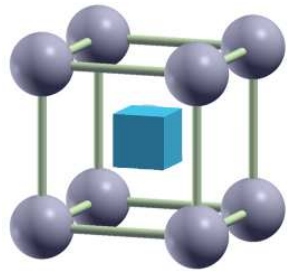


## □ Elimination on sinks

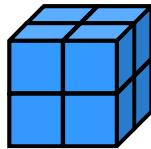
- Dislocation lines (network & loops),
- Grain-boundaries
- Free surfaces, voids, bubbles.

**Common feature:**  
thermally activated atom jumps ;  
slow evolution of the point defect population

# MORPHOLOGY OF DEFECT CLUSTERS IN METALS UNDER IRRADIATION

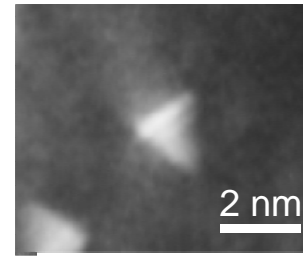


**3D**



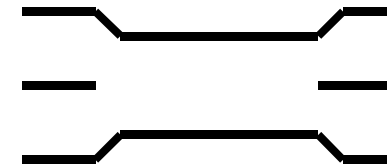
voids

**3D**

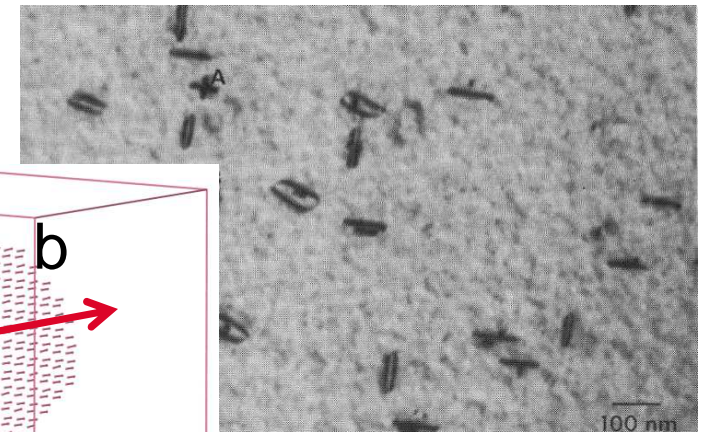
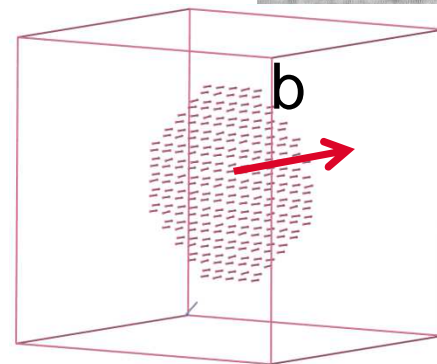
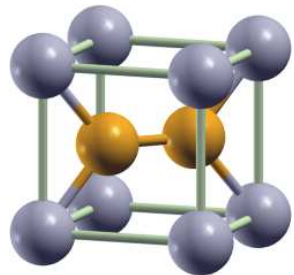


SFT

**2D**



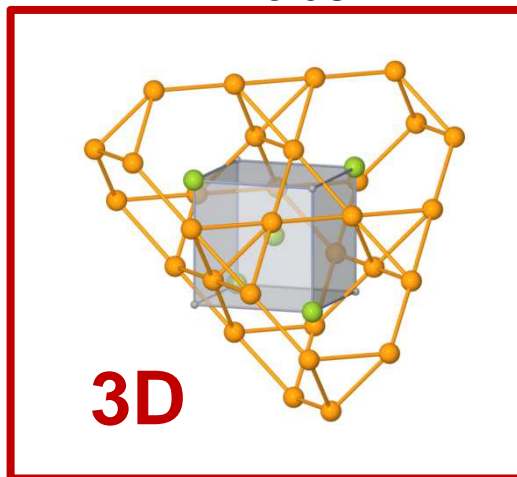
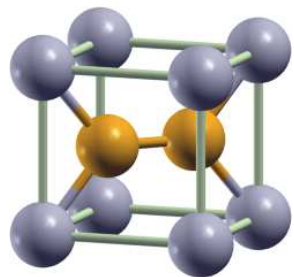
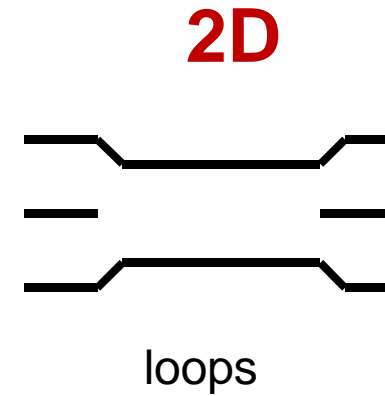
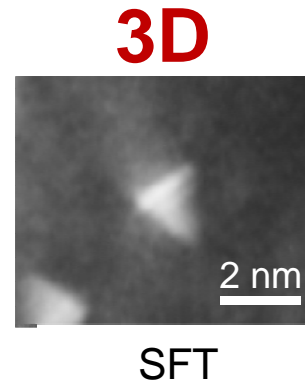
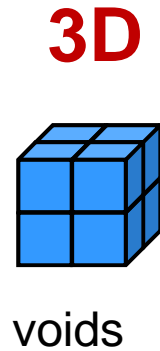
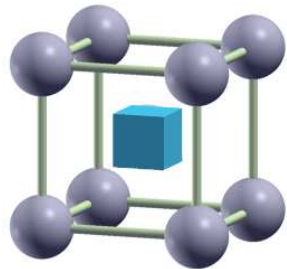
loops



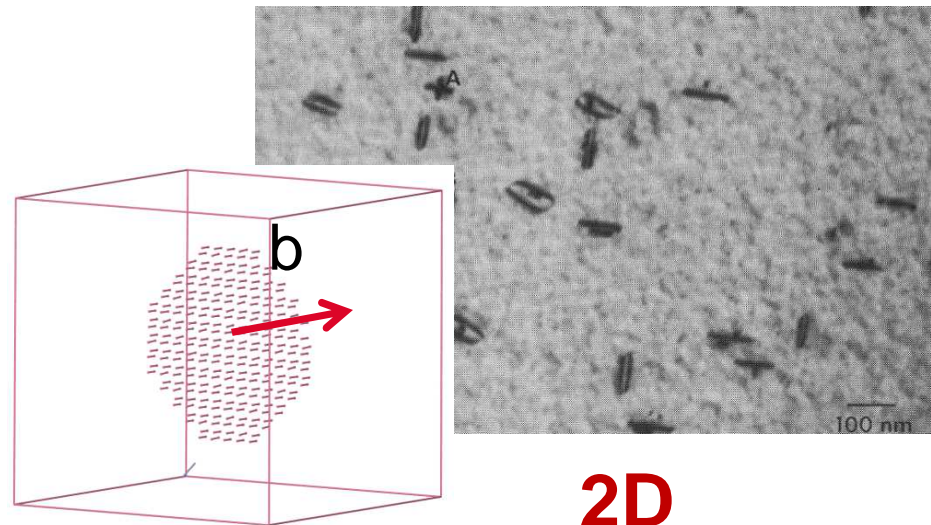
**2D**

# MORPHOLOGY OF DEFECT CLUSTERS IN METALS UNDER IRRADIATION

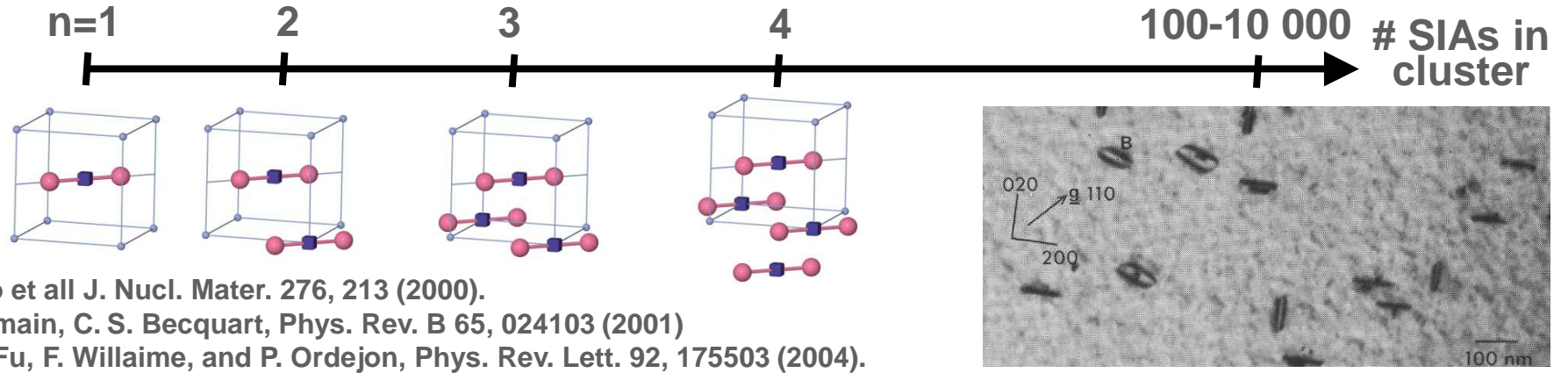
n=1 | 10 | 100-10 000 → # defects in cluster



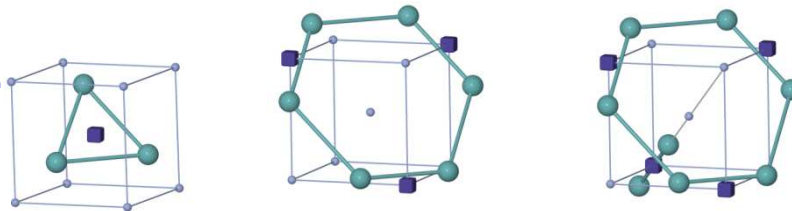
Prediction from atomistic simulations



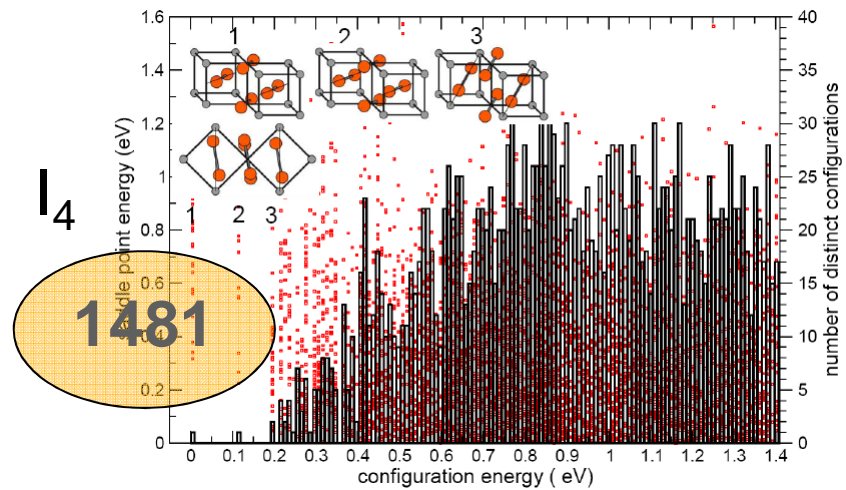
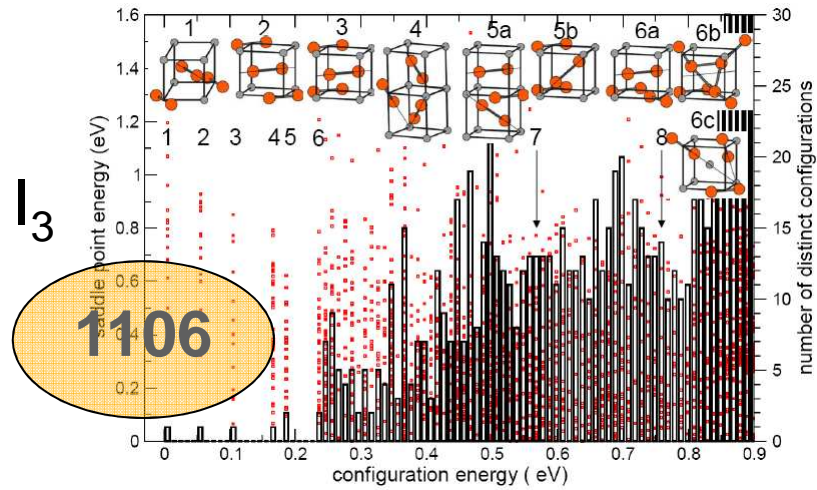
# New type of clusters



D.A Terentyev et al.  
 Phys. Rev. Lett.  
 100, 145503 (2008)



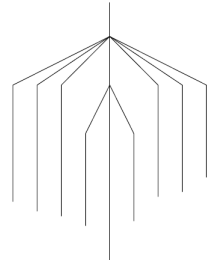
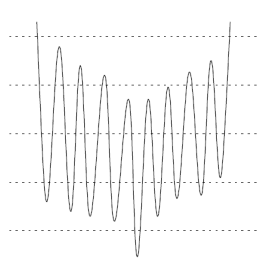




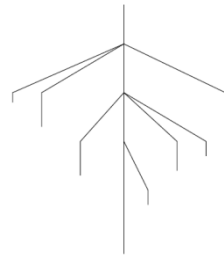
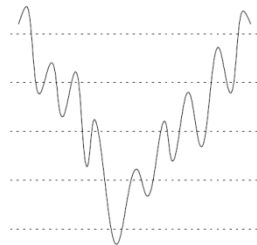
- many non-parallel configurations between  $I_3^{<110>}$  and  $I_3^{<111>}$
- quasi-continuum of states
- thousands of binding configurations for  $I_2$ ,  $I_3$  and  $I_4$ , respectively.

M.C. Marinica et al PRB 83 (2011) 094119

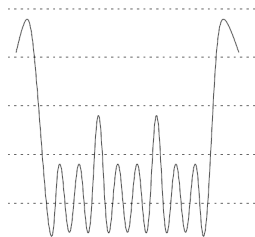
# Disconnectivity graph: archetypal energy landscape



**High downhill barriers, no well defined global minimum**



**Low downhill barriers and well defined global minimum**



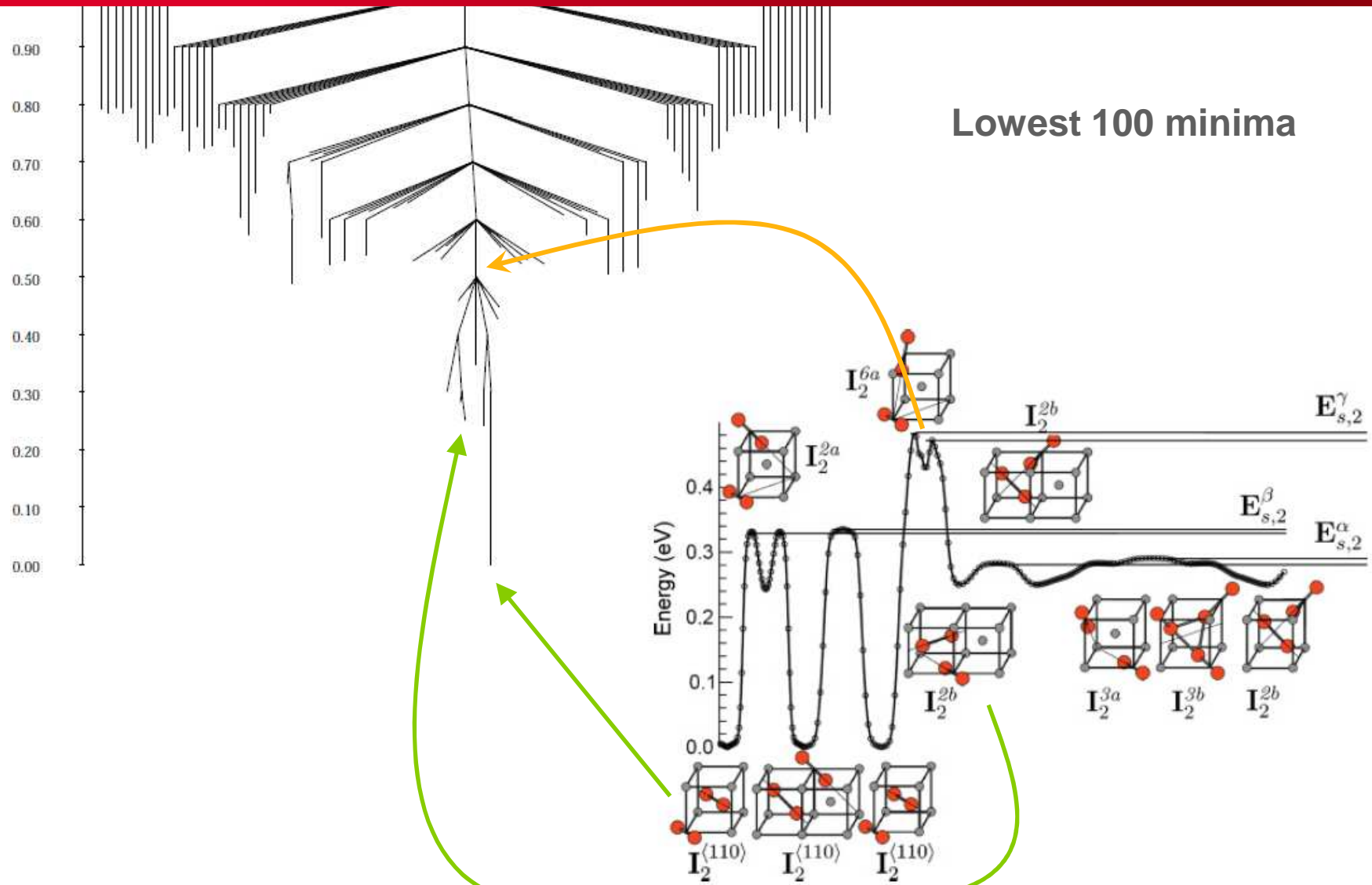
**'Rough' landscape, e.g. glasses**

O. M. Becker and M. Karplus, J. Chem. Phys. **106**, 1495 (1997)

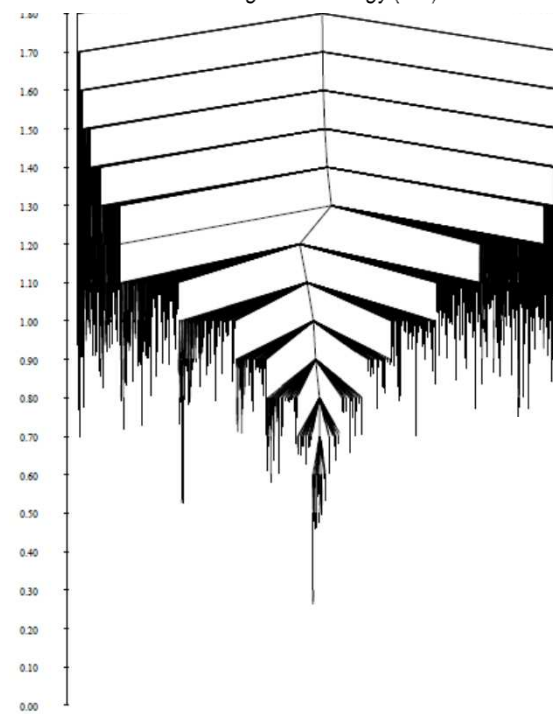
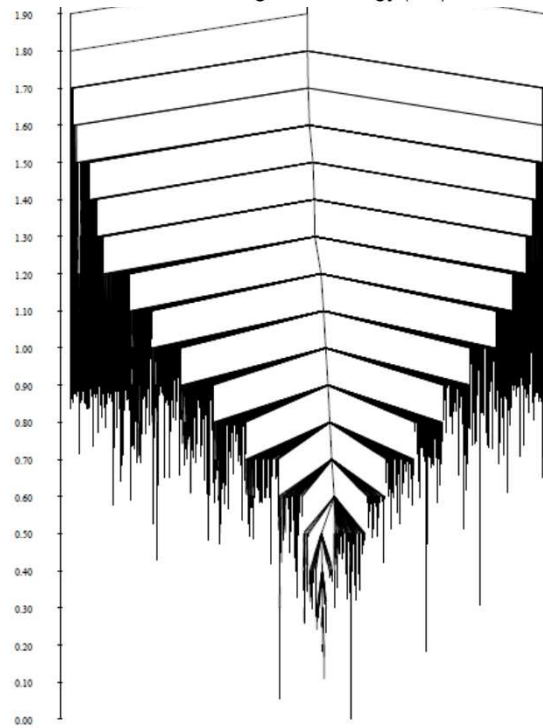
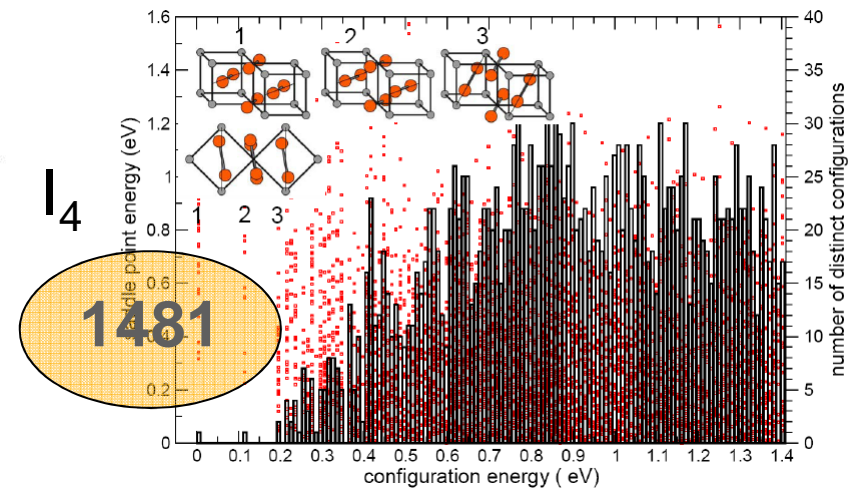
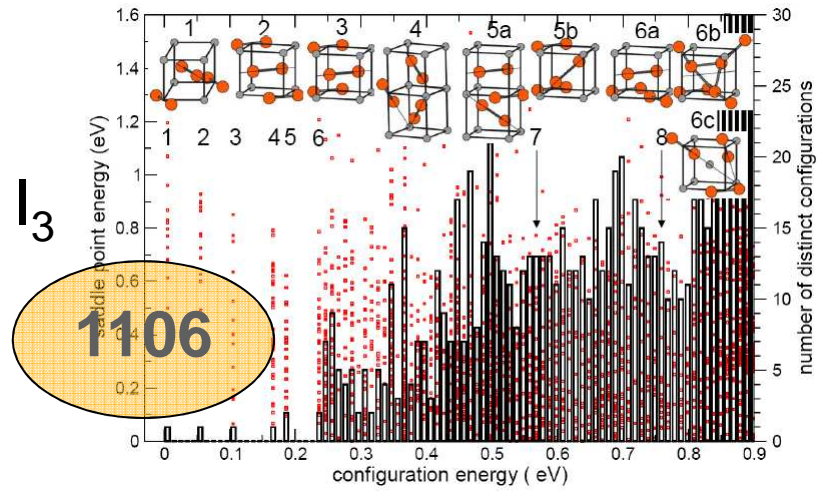
D. J. Wales, M. A. Miller, and T. Walsh, Nature **394**, 758 (1998)

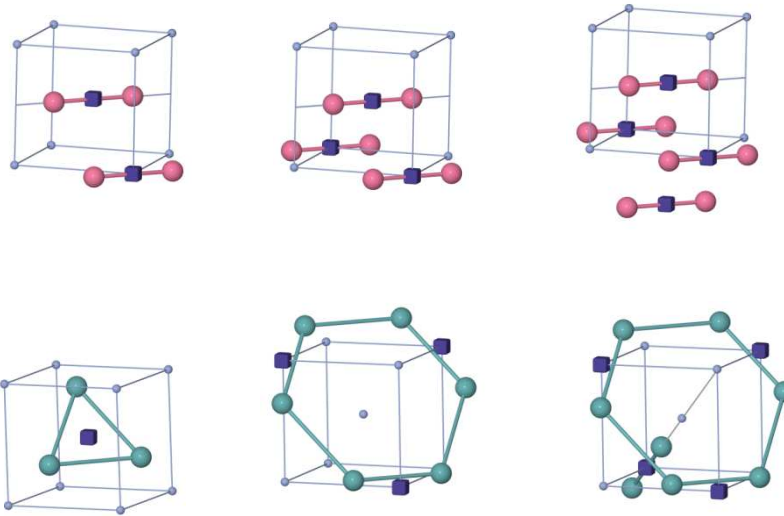
D. J. Wales, Energy Landscapes, Cambridge University Press

# Disconnectivity graph: Non-Parallel dumbbells in Fe



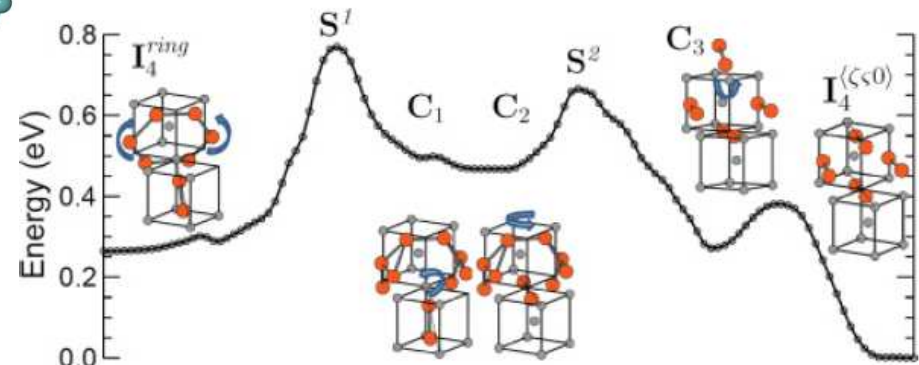






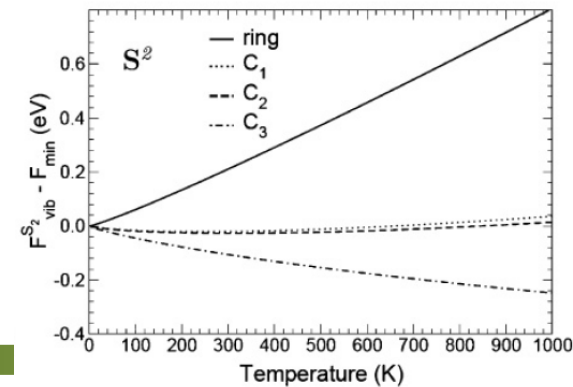
1. Self-trapped configurations (I4)
2. Stabilized at high temperature

D.A Terentyev et al Phys. Rev. Lett. 100, 145503 (2008)

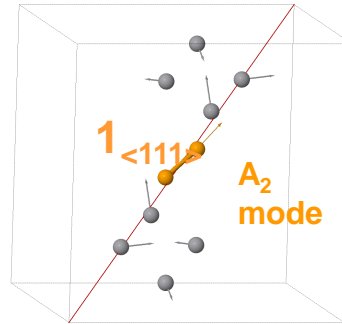


	$\Delta E_F$	$\Delta S_F$	$\Delta E_F$	$\Delta S_F$	$\Delta E_F$	$\Delta S_F$
EAM	0.31	6.5	0.79	10.0	0.25	12.9
SIESTA	-0.05		0.63		0.47	
VASP	-0.11		-0.06		0.07	

New configurations of I2, I3 and I4 become more stable above resp. 550 K, 920 K and 225K

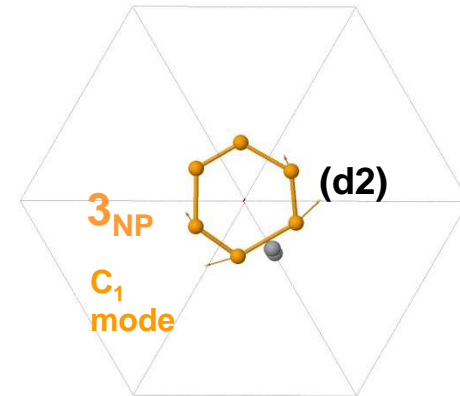
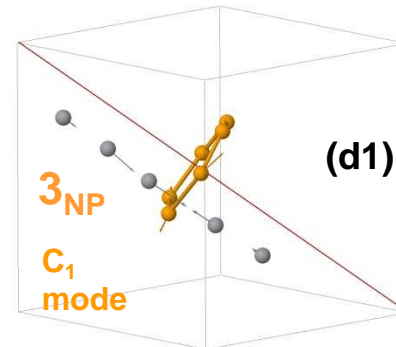
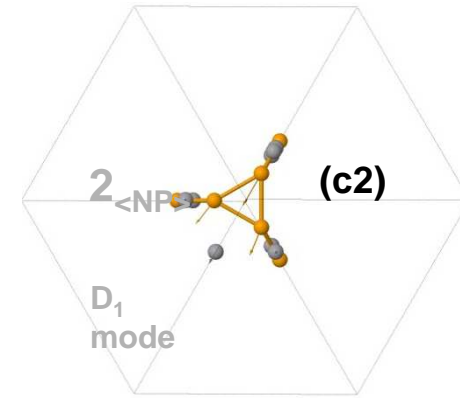
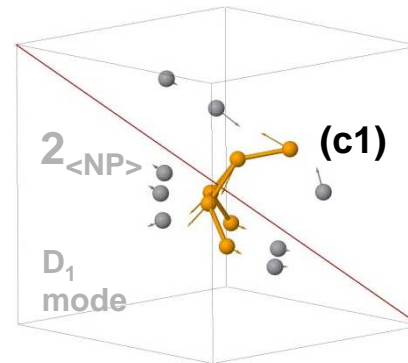
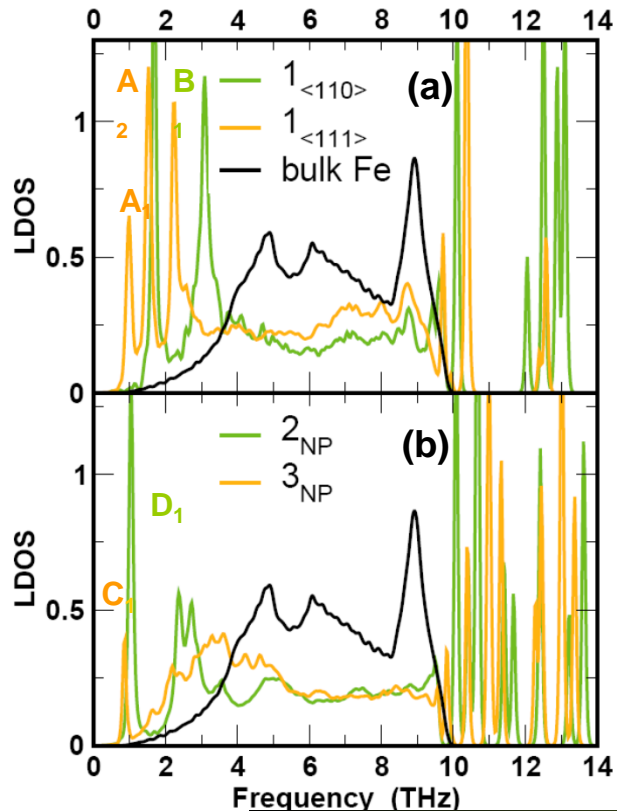


- For clarity we present a cell with only 128 atoms and
- the atoms where the localization of the mode ( $\sum_{\alpha} |\xi_{i\alpha,p}|^2$ ) is higher than a critical value ( 0.01 Å<sup>2</sup>)

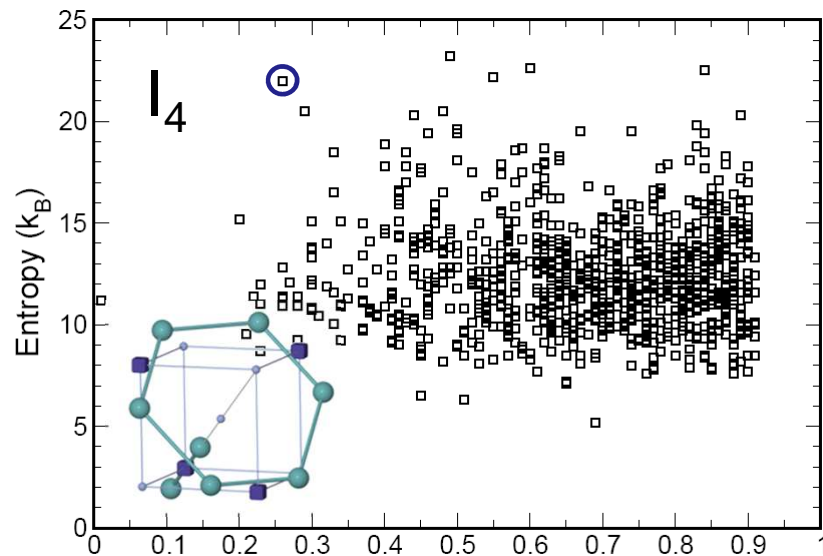
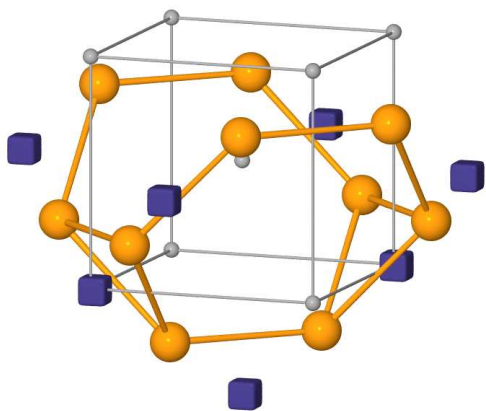
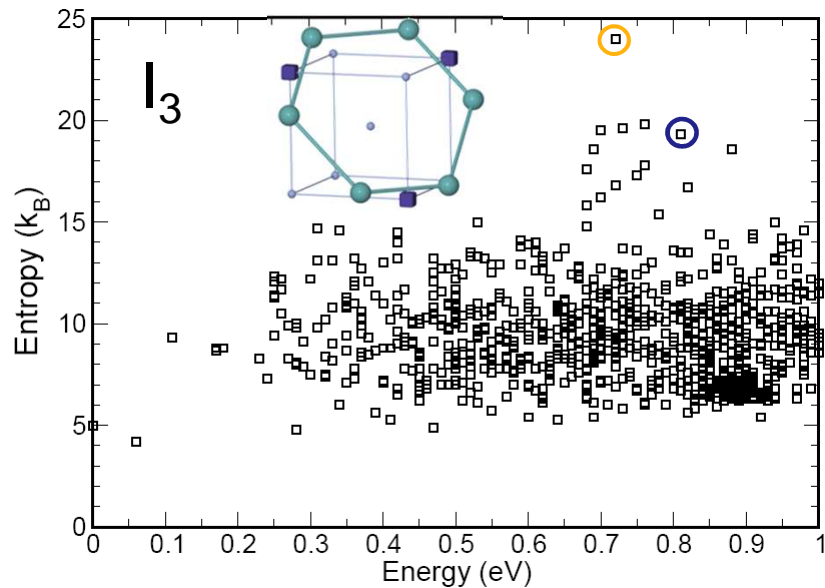
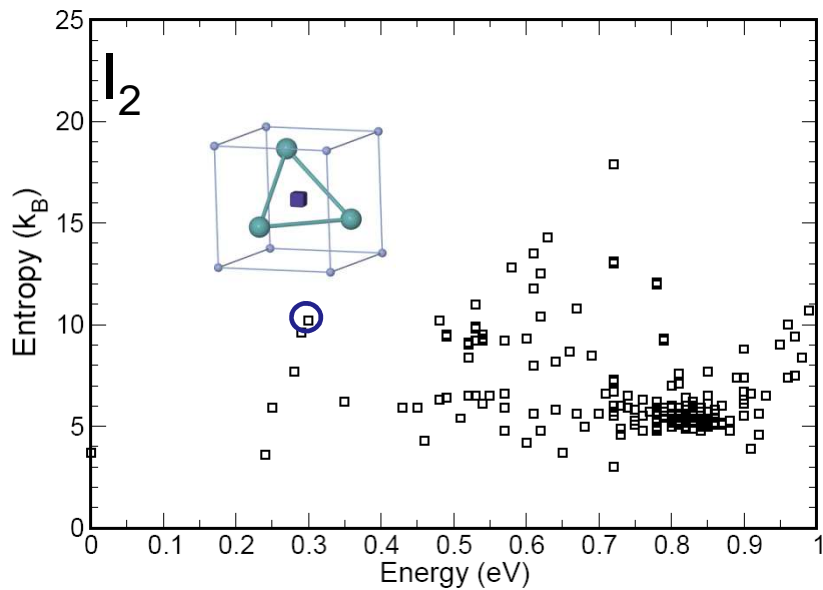


•  $\xi_{i\alpha,p}$  is the eigenvector associated to the  $i^{\text{th}}$  atom in the direction  $\alpha$  of the  $p^{\text{th}}$  mode.

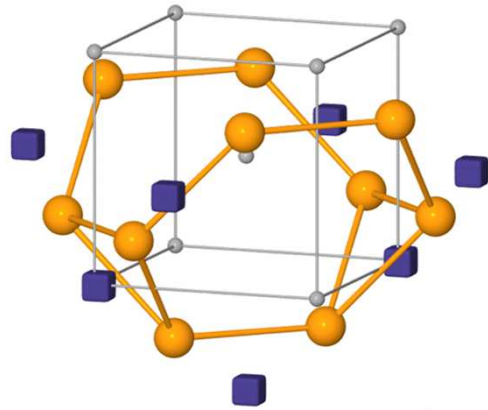
• The orange atoms are the defect atoms and gray atoms are the bulk atoms which fulfil the above localization criterion.



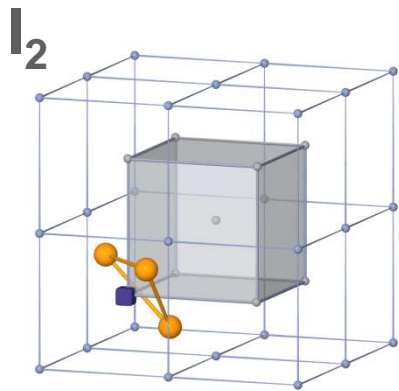
# Application to $I_n$ : ART+HA



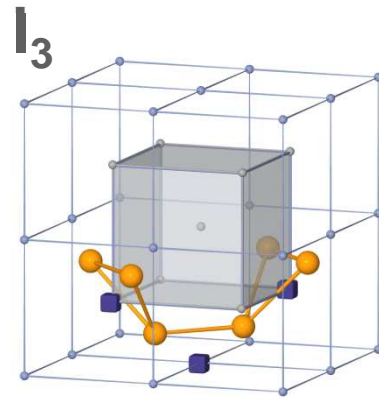
# Building a new type of interstitial clusters



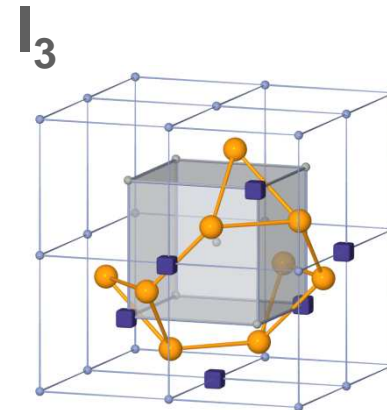
How to combine the triangle and ring defects ?



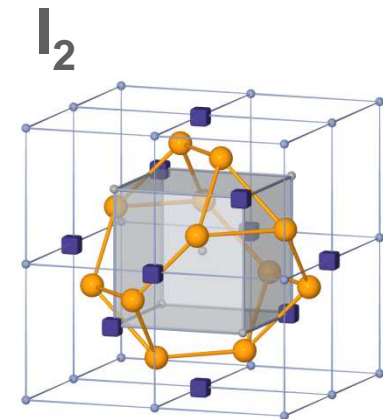
**3i - 1v**



**6i - 3v**



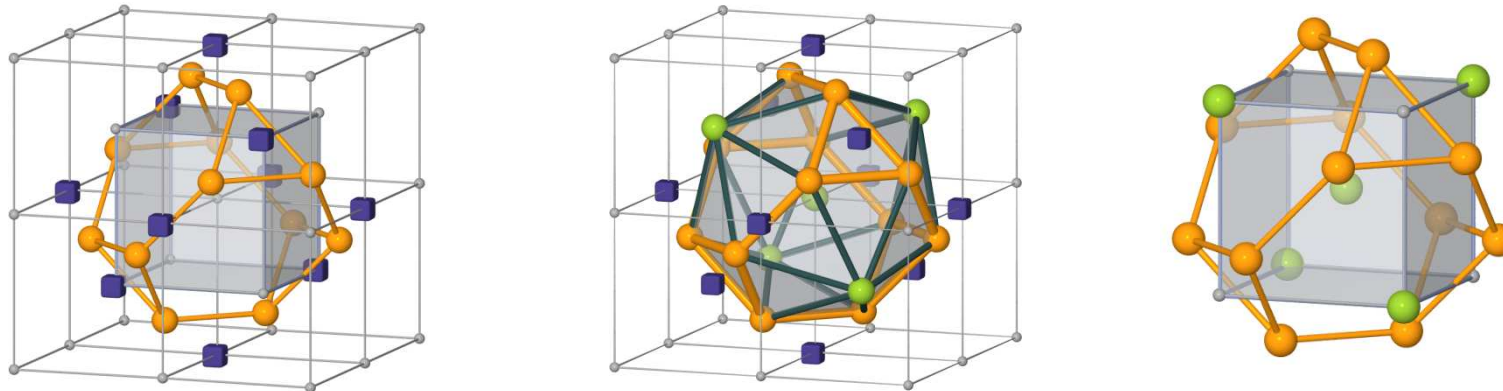
**9i - 6v**



**12i - 10v**

4 triangles  
4 rings

## Di-interstitial

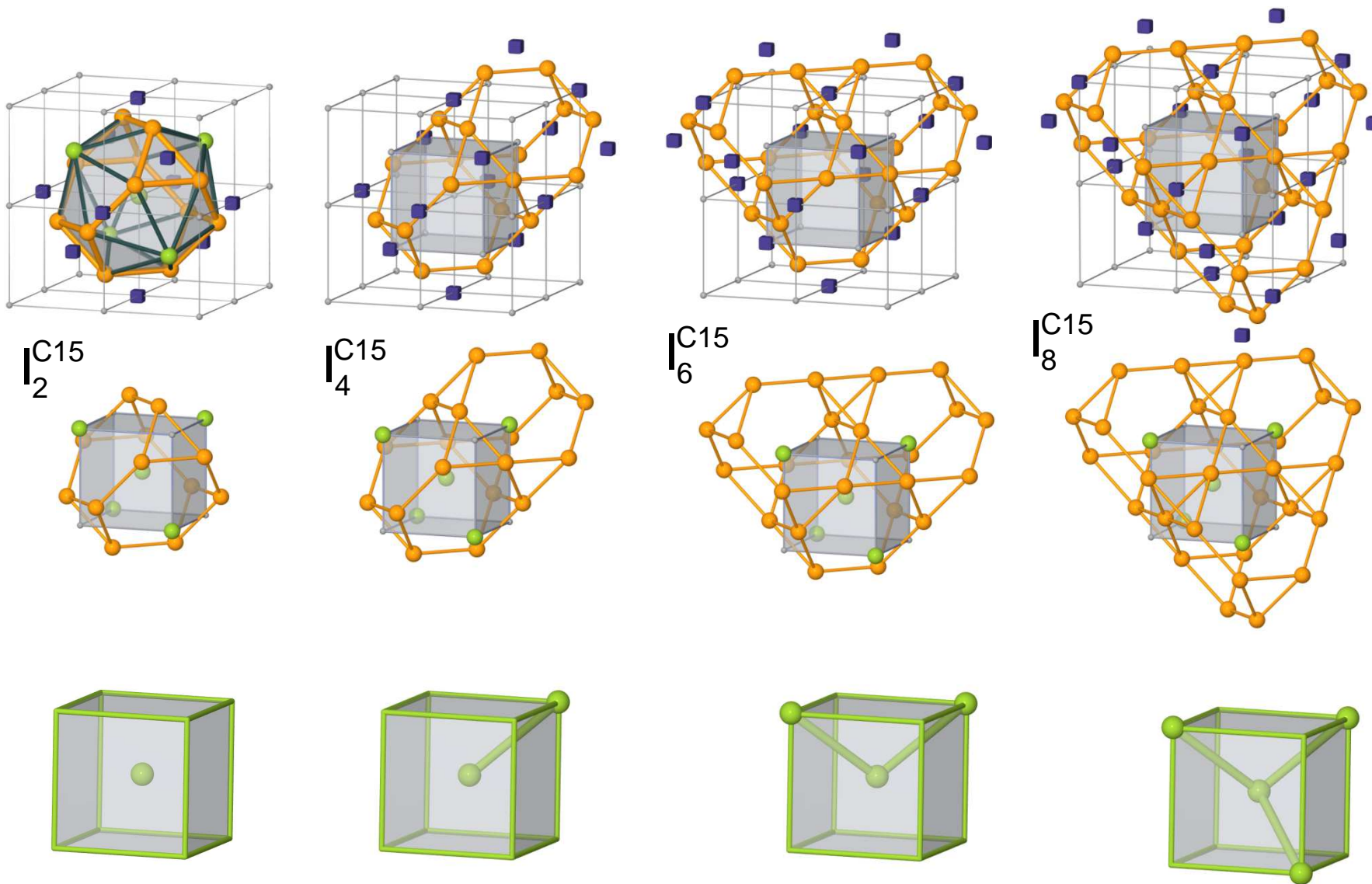


### Z16 Frank-Kasper polyhedra

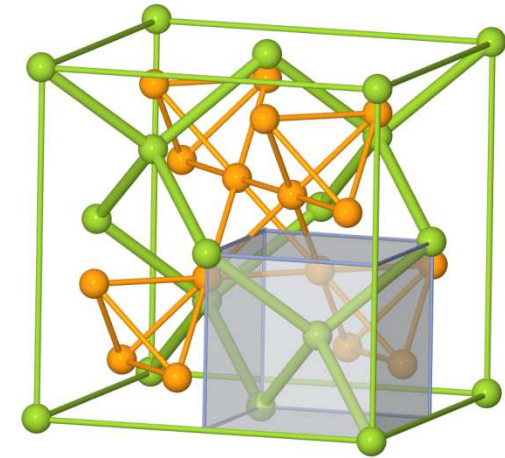
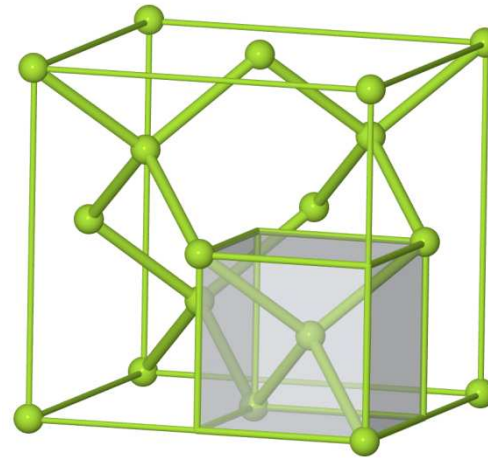
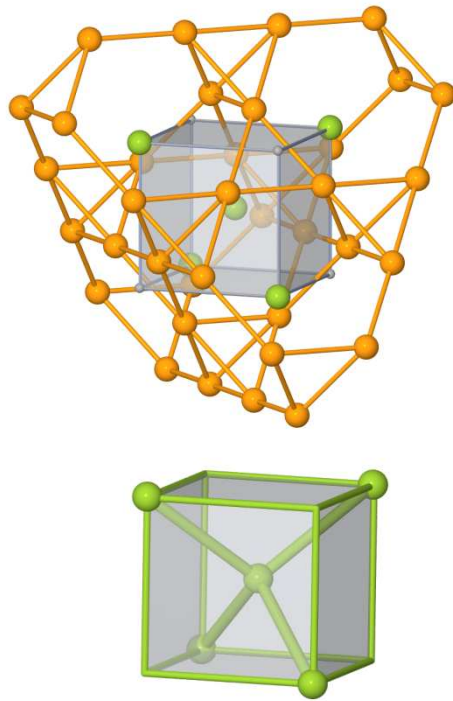
1. Low formation energy (ab initio): + 0.71 eV (compared to 110 cluster, 0.12 eV binding energy)
2. Very large antiferromagnetic moment:  $-33 \mu_B$  compared to the bulk !  
( $-8 \mu_B$  in the case of  $\langle 110 \rangle$  dumbbell)
3. Small formation volume (ab initio)
4. Large formation entropy (some empirical potential): + 24  $k_B$  (4  $k_B$  in the case of 110)



# Building larger clusters

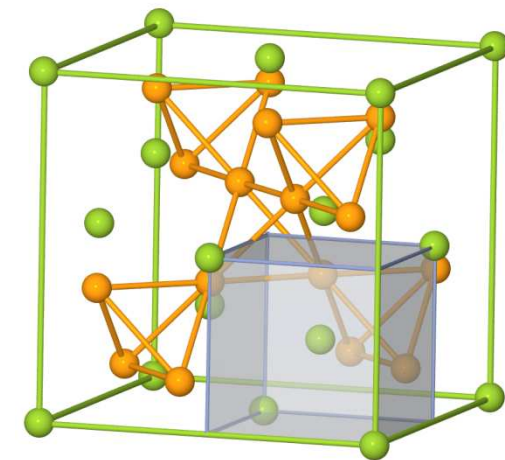


# Identification of the crystal structure: C15 Laves phase



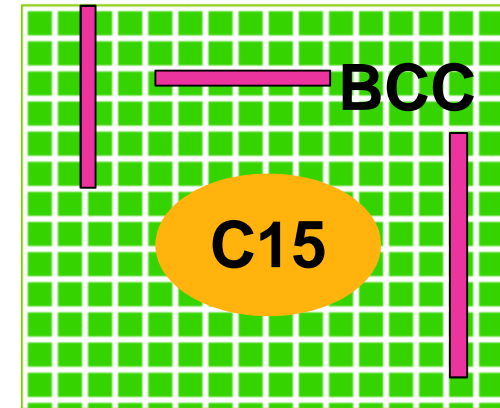
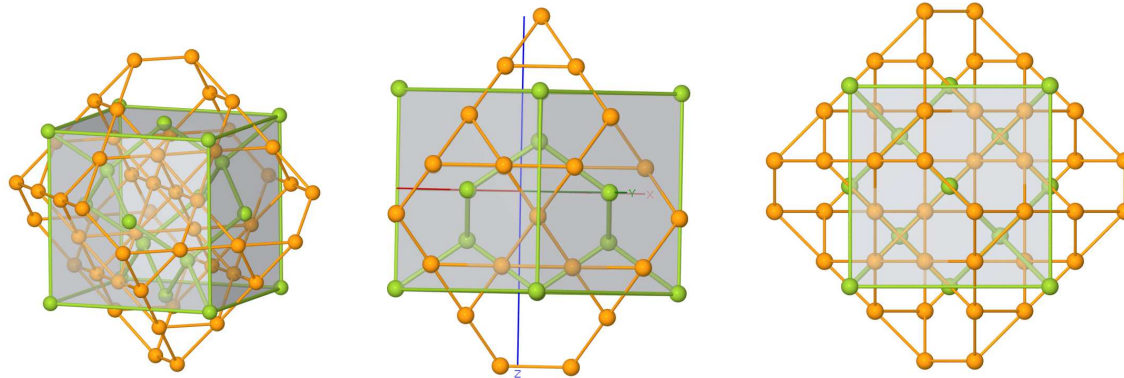
**Cubic Phase of Laves  
C15 or  $\text{MgCu}_2$**

$\text{MgCu}_2$





# Interstitial clusters in Cubic Phase of Laves C15 or MgCu<sub>2</sub>



- **C15** coherent with the **BCC** matrix
- **C15** have **3D** structure as opposed to the **2D** structure of dislocation loops

M.C. Marinica, F. Willaime, J.-P. Crocombette, Phys. Rev. Lett. (2012)

## Density Functional Theory

- *ab initio* electronic structure calculations
- PWSCF (Quantum Espresso) code
- 250 to 432 atom supercells



Formation energies of self-interstitial clusters (in all bcc metals)



## Empirical interatomic potential for Fe

- Embedded Atom Method type
- Parameters fitted to bulk and point defect properties (DFT database)



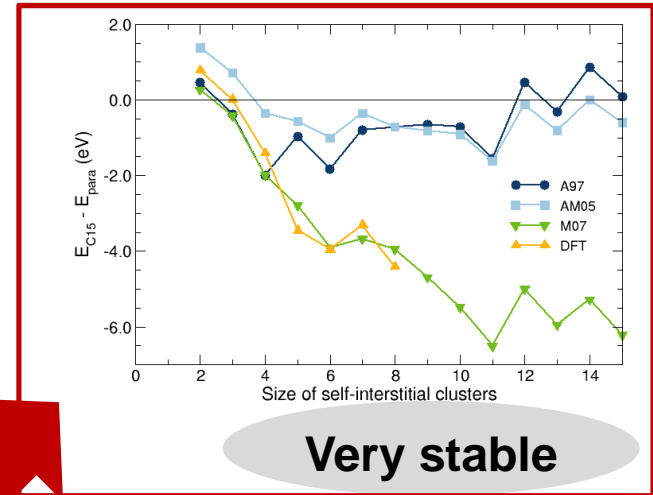
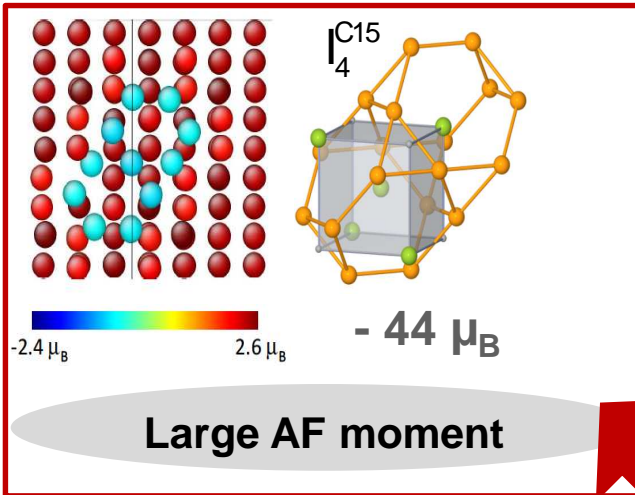
Cascade simulations  
(Molecular Dynamics)

Energy landscape of  
interstitial clusters  
(Activation Relaxation Technique)

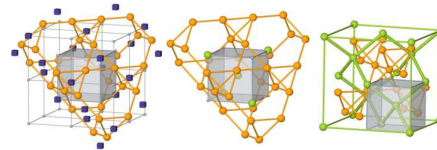


Representation  
(Disconnectivity graph)

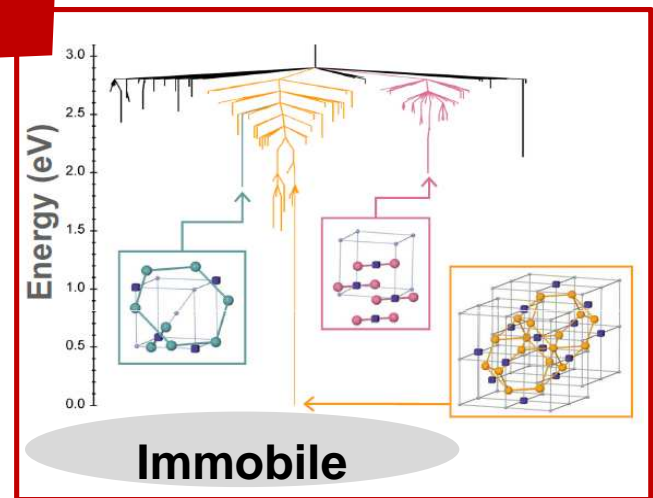
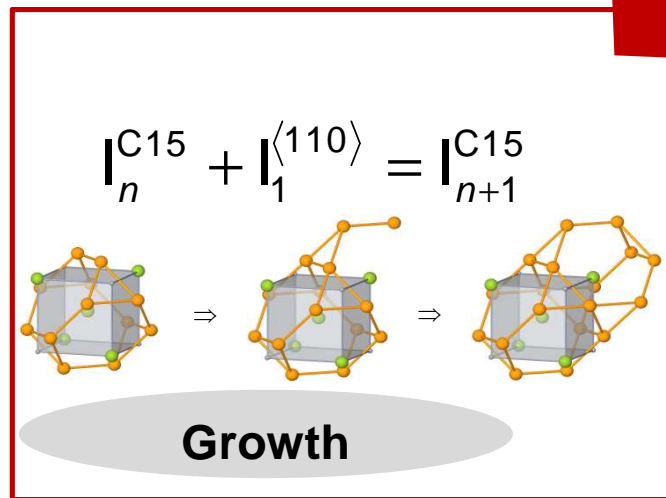
# Characteristics of C15 clusters



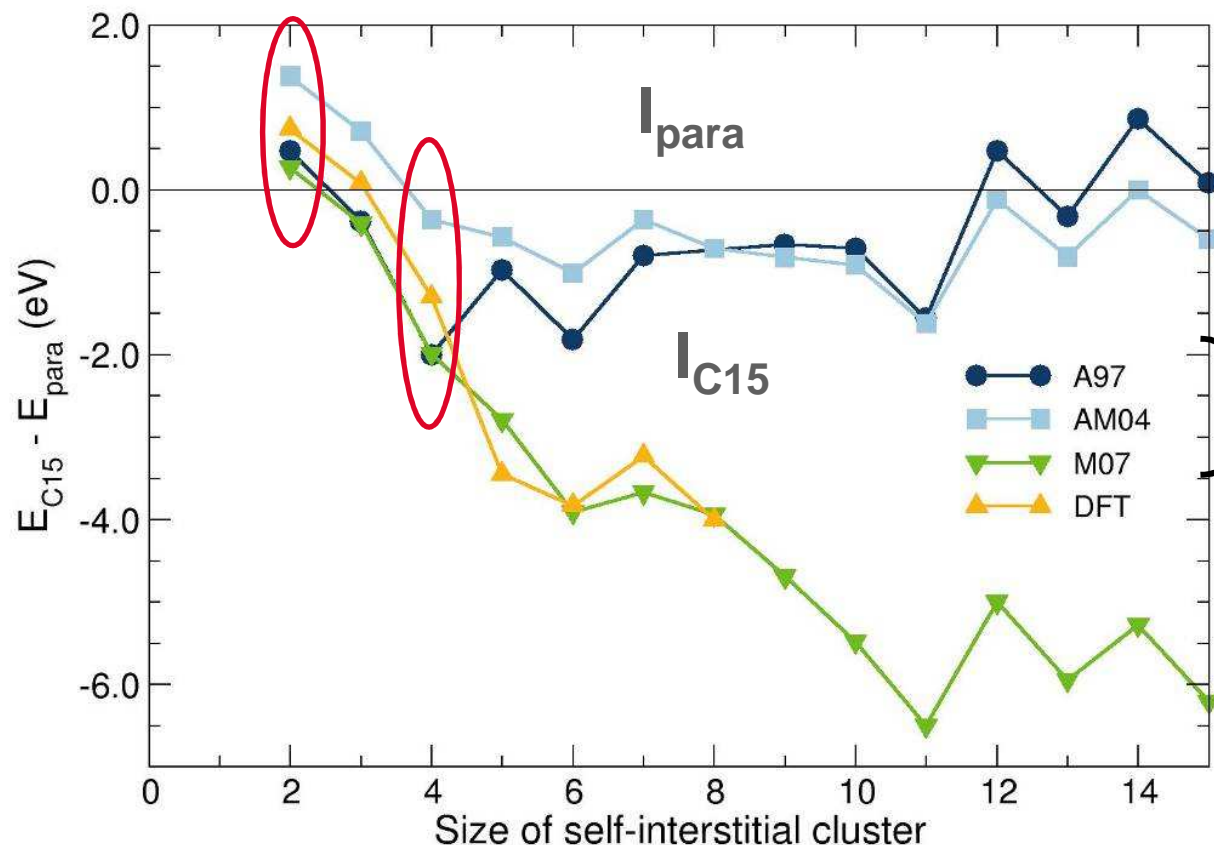
## C15 Clusters



1. Very stable
2. Immobile
3. **Can grow**
4. Large AF moment



Stability of C15 clusters against loops



M07: improved by including in the fit a large data base of DFT defect properties (no C15 clusters)

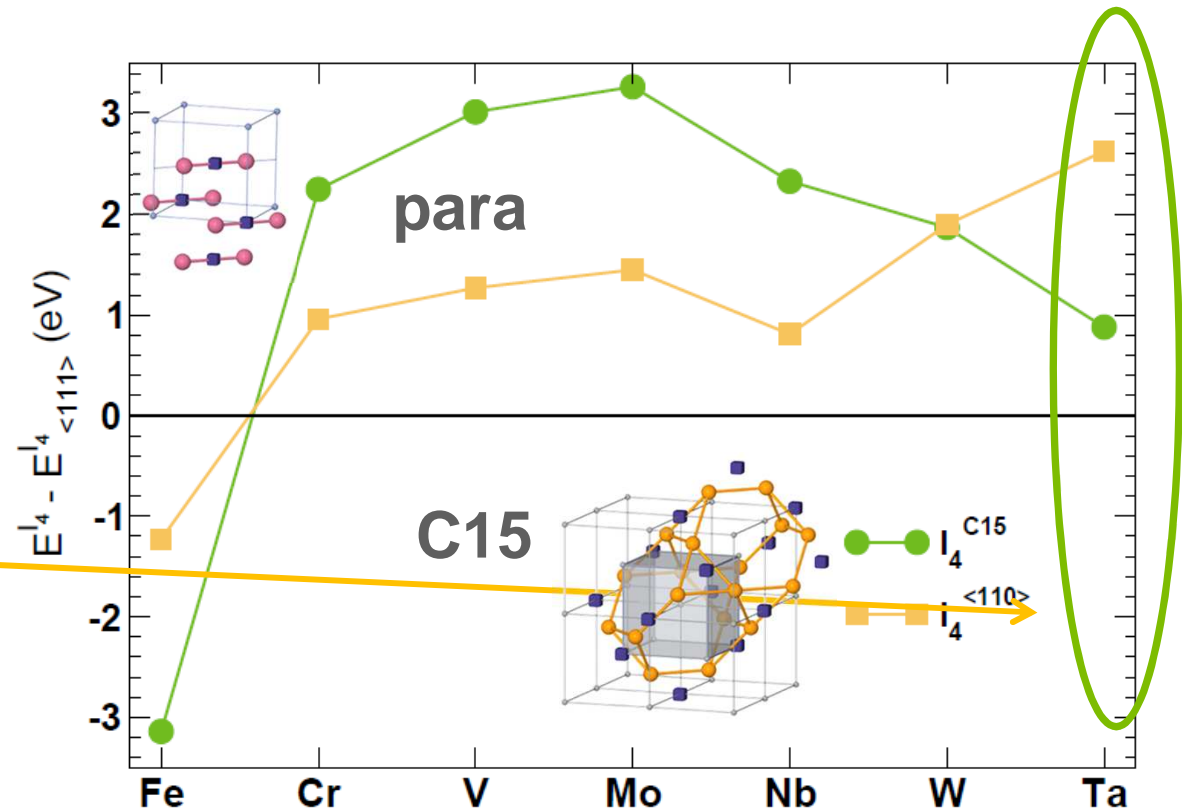
EAM potentials

\*Malerba et al. JNM 406, 19 (2010)  
with one typo corrected in:  
 MCM et al, PRL 108, 025501 (2012)

DFT calculations: PWSCF code, 250 atoms, tested against pseudopotential (USPP and PAW), semicore states, LDA/GGA, cell size

1. Not in other bcc metals

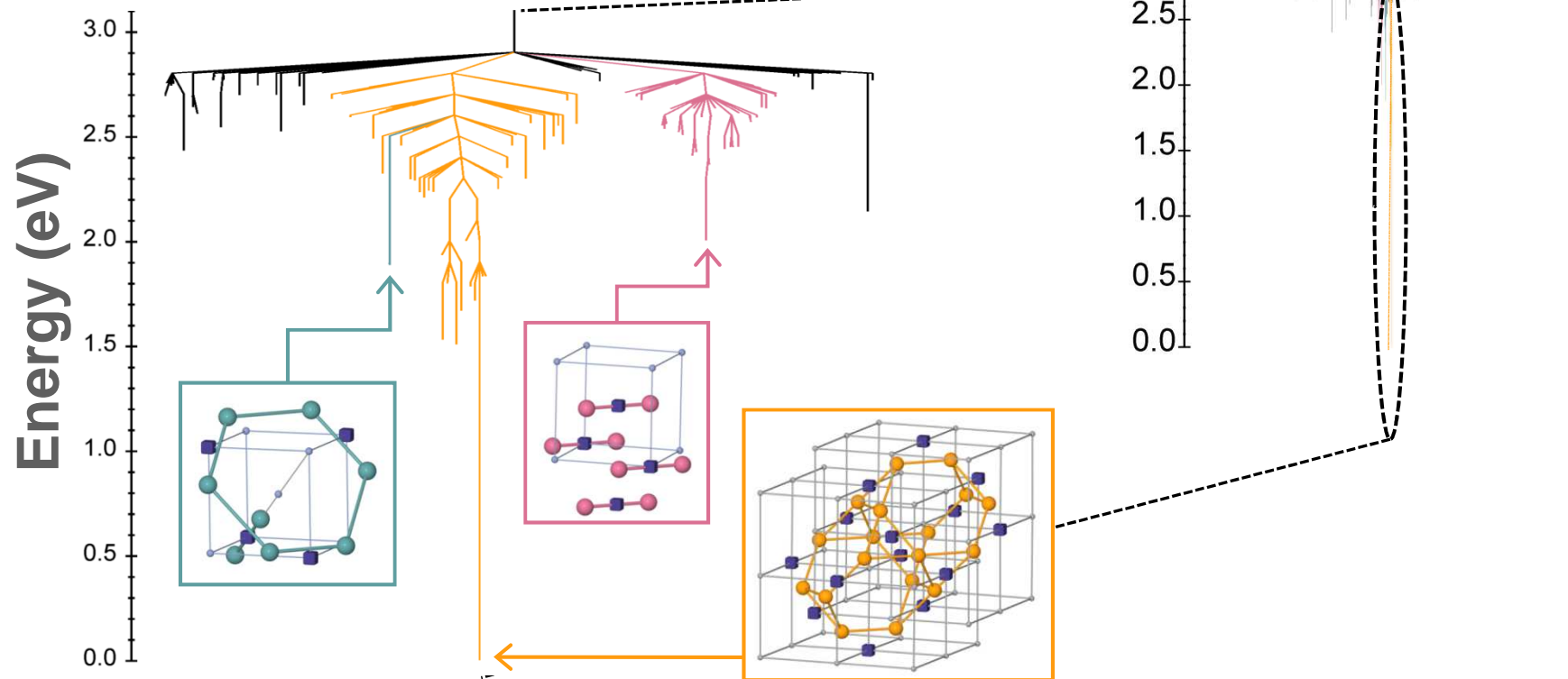
2. Ta has interesting property



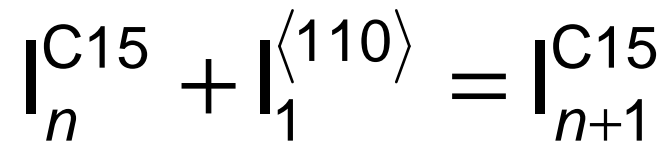
- M07 EAM potential for Fe
- Activation Relaxation Technique (ART)
- disconnectivity graph representation

O. M. Becker and M. Karplus, J. Chem. Phys. **106**, 1495 (1997)

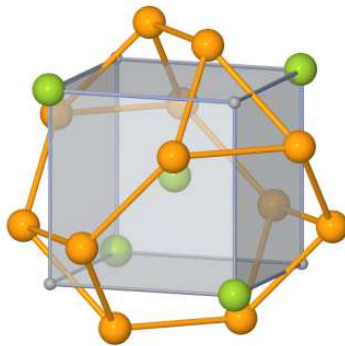
D. J. Wales, M. A. Miller, and T. Walsh, Nature **394**, 758 (1998)



1. Two basins: **C15** and **parallel dumbbells** ; confirms that C15 is the lowest energy structure
2. shows that it is immobile (confirmed by MD simulation over  $\mu\text{s}$ )
3. . **Older clusters** identify a particular branch of the C15 basin

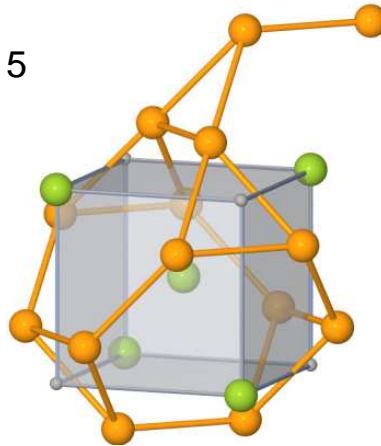


$|C15_2$



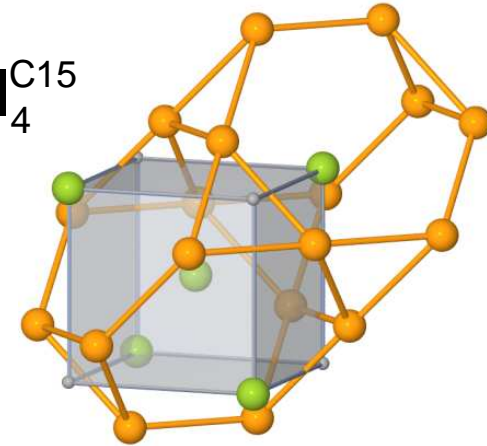
$\Rightarrow$

$|C15_3$



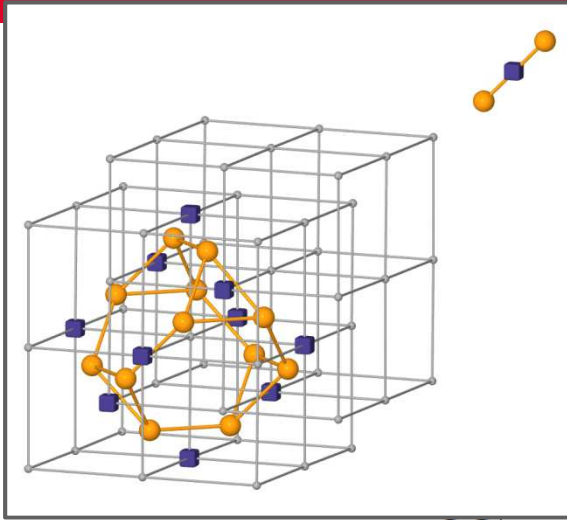
$\Rightarrow$

$|C15_4$



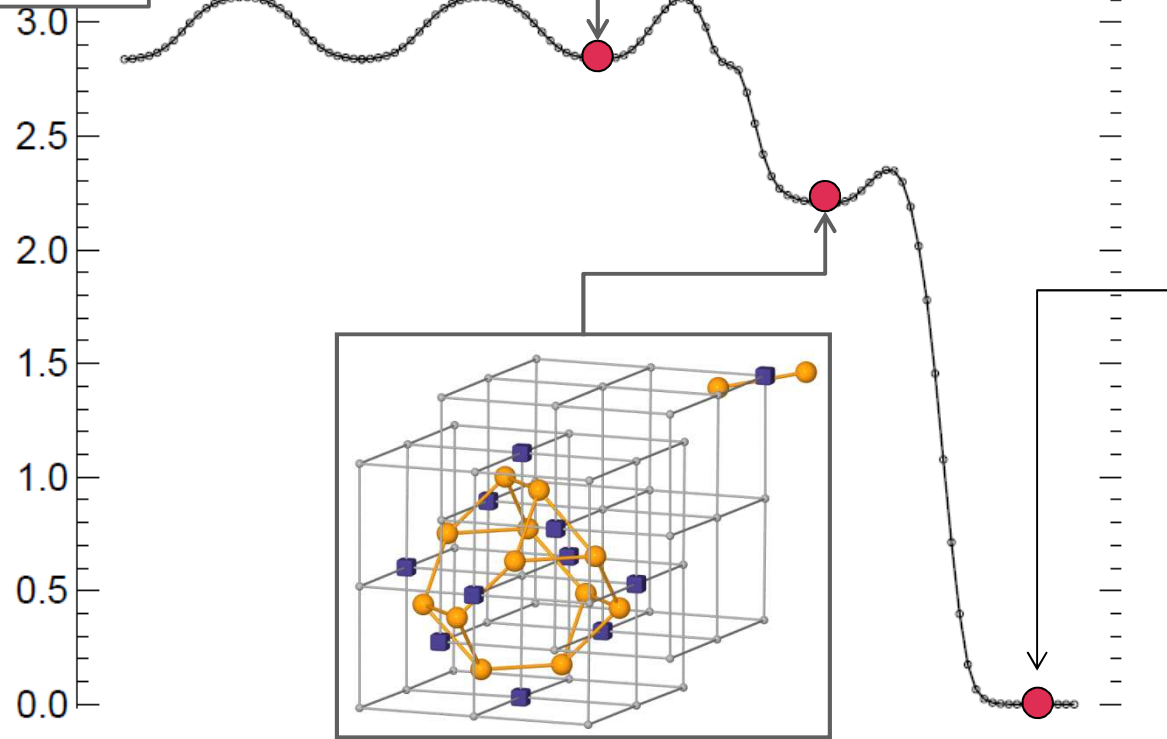
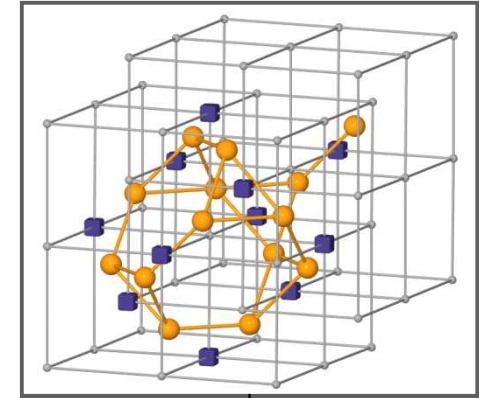
- ARTn





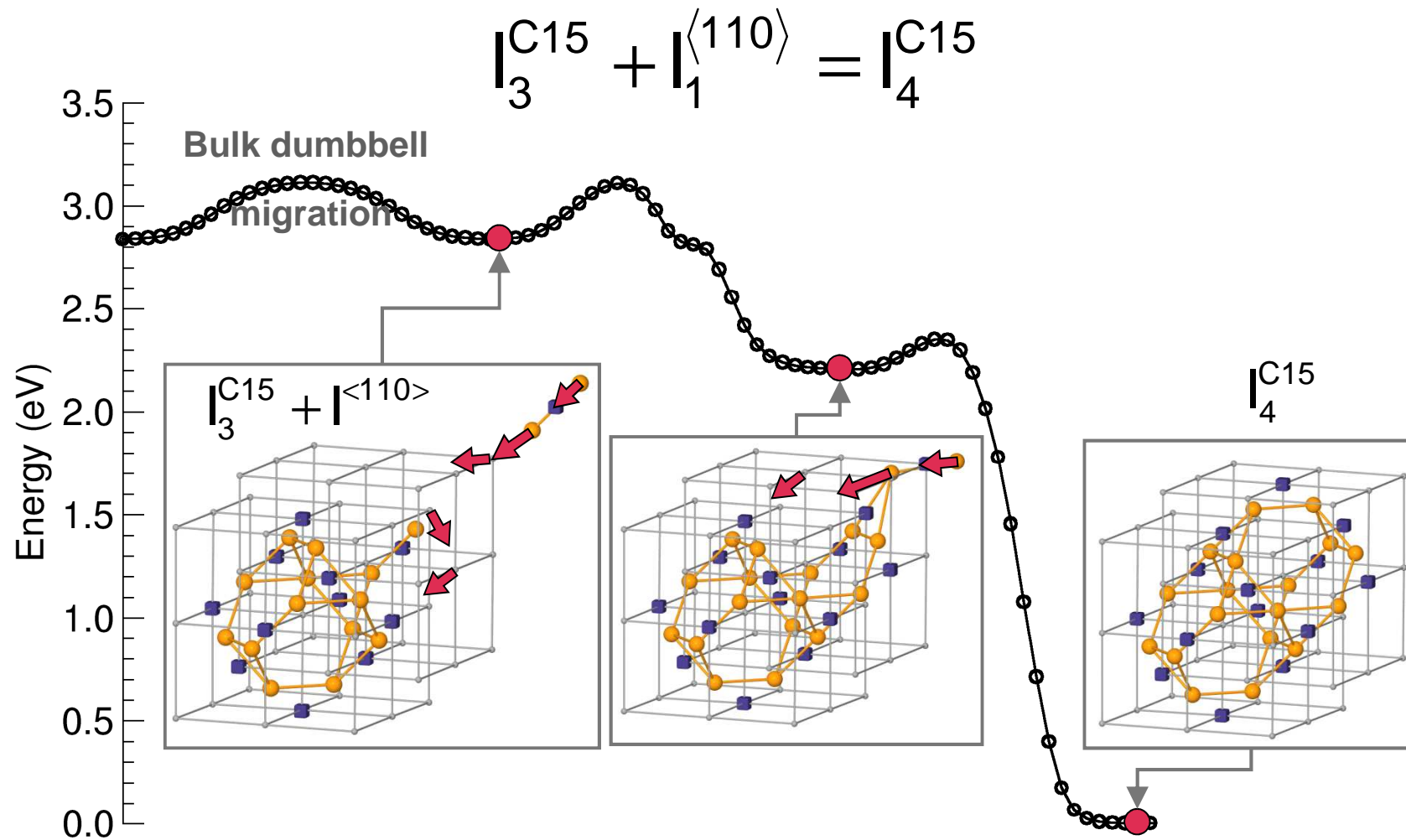
$$\left| C_{2}^{15} + \left\langle 110 \right\rangle = \left| C_{3}^{15} \right.$$

Bulk dumbbell migration



Doesn't exceed the barrier for <110> dumbbell migration





Doesn't exceed the barrier for <110> dumbbell migration

## Using a method for systematic search in the energy landscape we have predicted a 3D crystalline structure for self-interstitial clusters in bcc metals

- Very low energy structures
- Can grow by capturing  $\langle 110 \rangle$  dumbbells
- Have large antiferromagnetic moments in Fe
- Are immobile

