

# Low-dimensional magnetism in DFT



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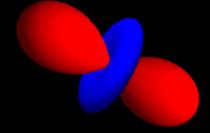
**A. Saúl**

CiNAM UMR 7325 CNRS, Aix-Marseille Université



**G. Rousse, Y. Klein**

IMPMC UMR 7590 CNRS, Université Pierre et Marie Curie



## People working on modelisation in the lab :

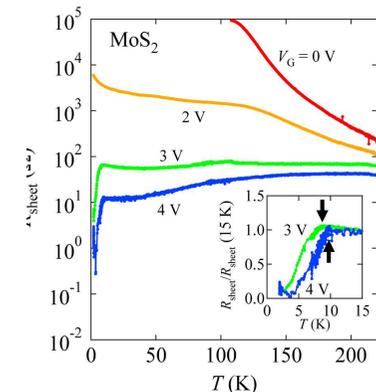
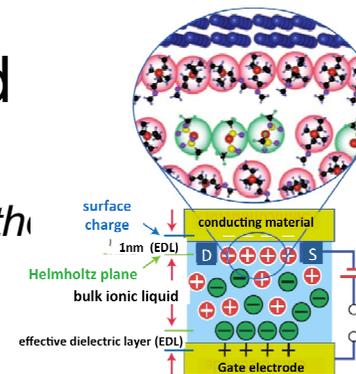
Ch. Brouder (DR-CNRS sec. 3)  
 D. Cabaret (MdC-Paris 6)  
 M. Calandra (DR-CNRS)  
 M. Casula (CR-CNRS, sec. 3)  
 M. Lazzeri (CR-CNRS)  
 F. Mauri (DR-CNRS)  
 L. Paulatto (IR-CNRS)

M. Saitta (PR-Paris 6)  
 G. Ferlat (MdC-Paris 6)  
 M. Blanchard (CR1-CNRS sec. 18)  
 E. Balan (CR-IRD)  
 A. Juhin (CR1-CNRS sec. 5)  
 M.A. Arrio (CR1-CNRS sec. 5)  
 G. Radtke (CR-CNRS, sec. 15)

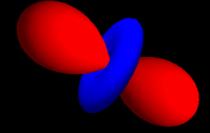
## Quantum theory of materials with and beyond DFT

*Prediction of materials properties based on ab initio meth*

- ▶ 2D materials beyond graphene
- ▶ Molecular superconductivity
- ▶ Ab-initio theory of charge and heat transport in nanostructures
- ▶ Beyond DFT : Quantum Monte-Carlo



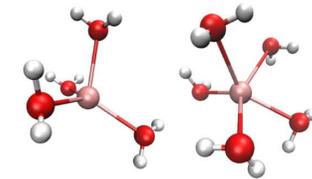
▲ **MoS<sub>2</sub> becomes metallic and superconducting once doped by K or Rb intercalation (T<sub>c</sub>~10 K) or in a FET [Taniguchi et al., APL 101, 042603 (2012)]**



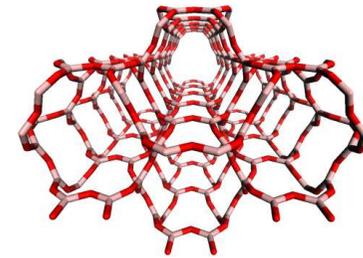
## Disordered matter

*Microscopic structure of liquids and glasses in high-pressure/high-temperature conditions with ab initio molecular dynamics*

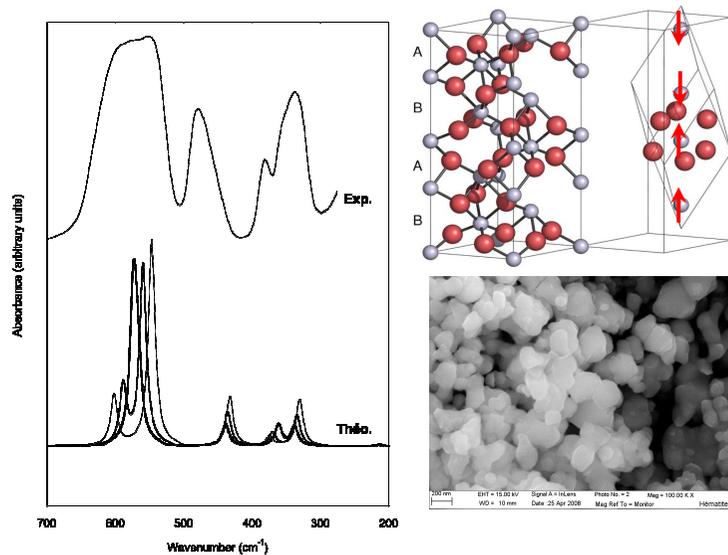
- ▶ structure of simple glasses and crystals ( $B_2O_3$ ) [Nature Mat. 2012]
- ▶ polyamorphism under high temp./pressure in  $GeO_2$  [PRB 2012]
- ▶ nucleation phenomena in glasses
- ▶ ionic solvation in supercritical fluids [J. Chem. Phys. 2011]



▲  $Ag^+$  ions in supercritical water



▲ Prediction of a porous  $B_2O_3$  polymorph.

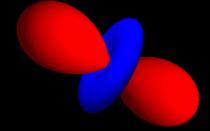


▲ IR spectra in  $\alpha-Fe_2O_3$  and SEM image of the sample.

## Geomaterials

*Vibrational and isotopic properties in minerals and molecules in DFT*

- ▶ IR and Raman modeling in  $\alpha-Fe_2O_3$  [Am. Min. 2008]
- ▶ Isotopic fractionation in cosmochemical processes [Earth Plan. Sci. Lett. 2012]

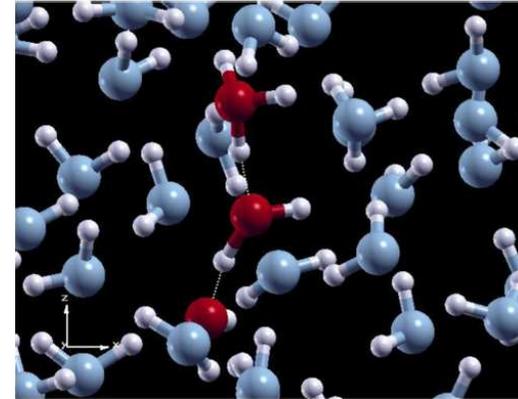


## Condensed matter under extreme conditions

*Theory of matter under extreme conditions*

*Ab initio calculation of phase diagrams, Raman spectra modeling, molecular dynamics*

- ▶ Molecular dynamics of dissociation of Water under an electric field [PRL 2012]
- ▶ Pressure-induced polyamorphism in salty water [PRL 2011]
- ▶ Pressure-induced transformations of  $\text{AlPO}_4$  [Nature Mat. 2007]

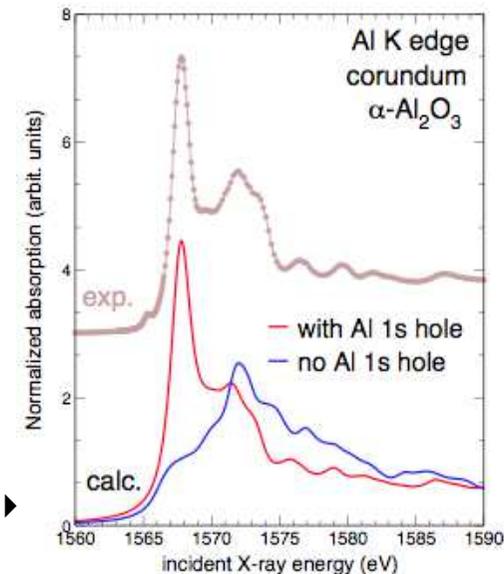


▶ Snapshot of a typical dissociation-diffusion mechanism in water under an electric field

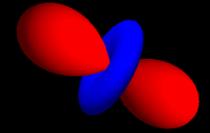
## Core-level X-ray spectroscopies

*XANES, XMCD, RIXS-MCD*

- ▶ one-electron (Xspectra in Quantum Espresso)
- ▶ multi-electronic (ligand-field multiplets)



Core-hole effect on calculated Al-K edge in  $\alpha\text{-Al}_2\text{O}_3$ . ▶

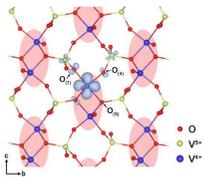


To interpret experiments, systems of localized moments are often modeled with Heisenberg Hamiltonians :

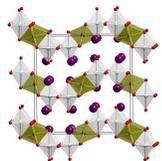
$$\hat{H} = \hat{H}_0 + \sum_{i>j} J_{ij} \hat{\mathbf{S}}_i \hat{\mathbf{S}}_j$$

The determination of the dominant couplings based on geometry is often difficult, sometimes misleading ...

Two surprising systems based on magnetic dimers revealed by DFT :

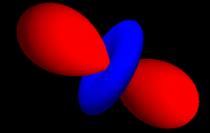


**CsV<sub>2</sub>O<sub>5</sub> :**  
**where the magnetic dimers are NOT the structural dimers**

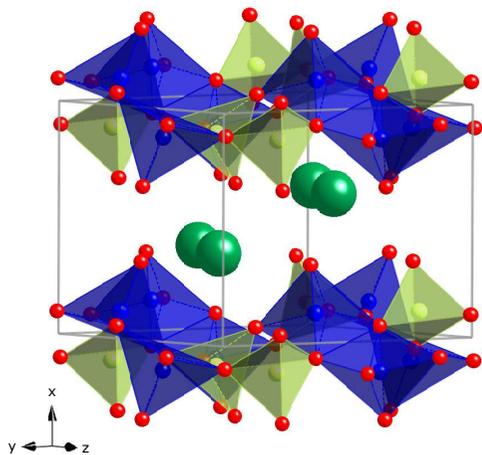


**Ba<sub>4</sub>Ru<sub>3</sub>O<sub>10</sub> :**  
**where structural trimers behave as magnetic dimers**

# Localized magnetism in $\text{CsV}_2\text{O}_5$

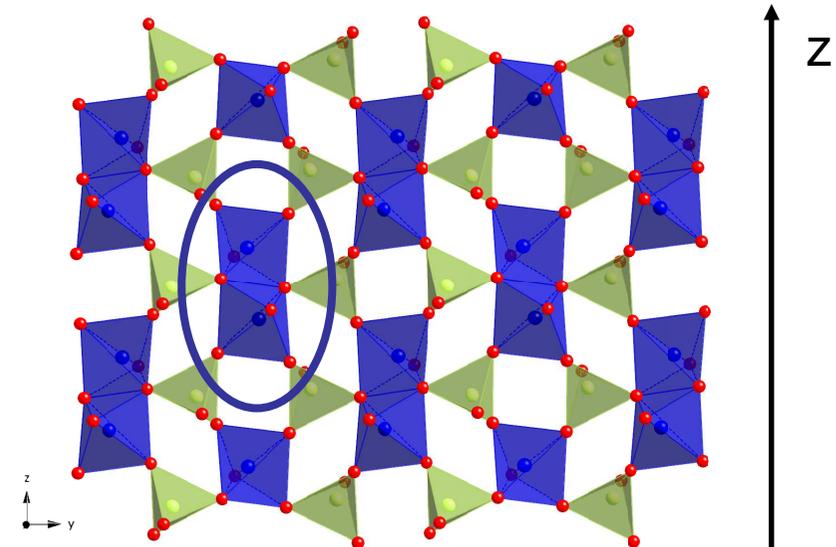


Monoclinic structure  $P2_1/c$   
Alternating layers of Cs and  $\text{V}_2\text{O}_5$

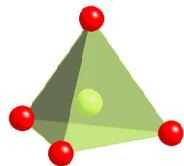


←  $\text{V}_2\text{O}_5$   
← Cs  
←  $\text{V}_2\text{O}_5$

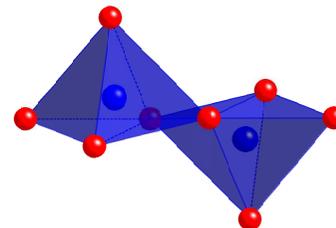
Structure of the layers



Two different V atoms in the layers

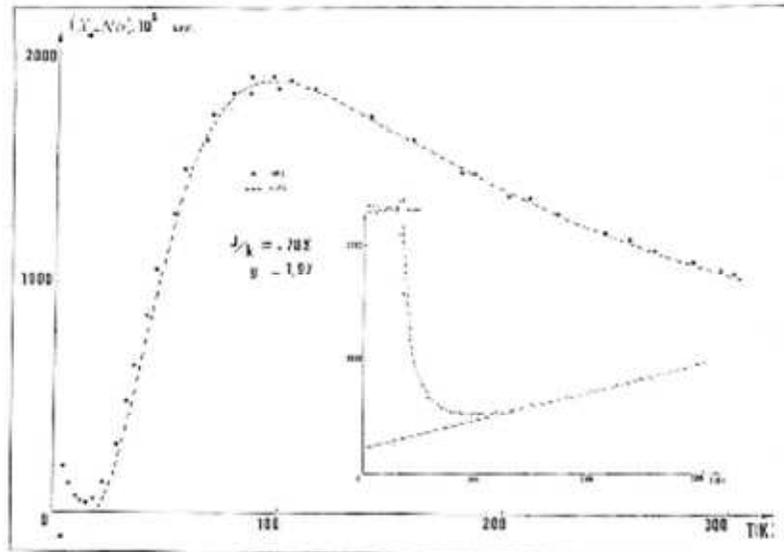
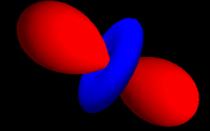


$\text{V}^{5+}$  in tetrahedra  
→ non-magnetic ( $3d^0$ )



$\text{V}^{4+}$  in distorted square  
pyramids sharing edges  
→ magnetic ( $3d^1$ )

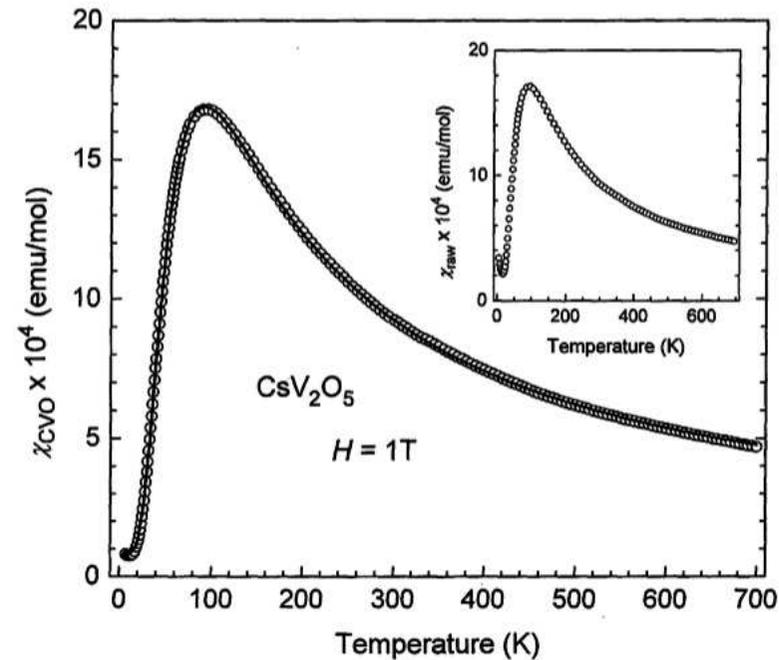
# Confirmation : the magnetic susceptibility



J. Mur and J. Darriet  
*C. R. Acad. Sc. Paris* **300** (1985) 599

➡ Isolated dimers :  $J = 156$  K

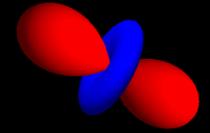
➡ Does DFT confirm this picture ?



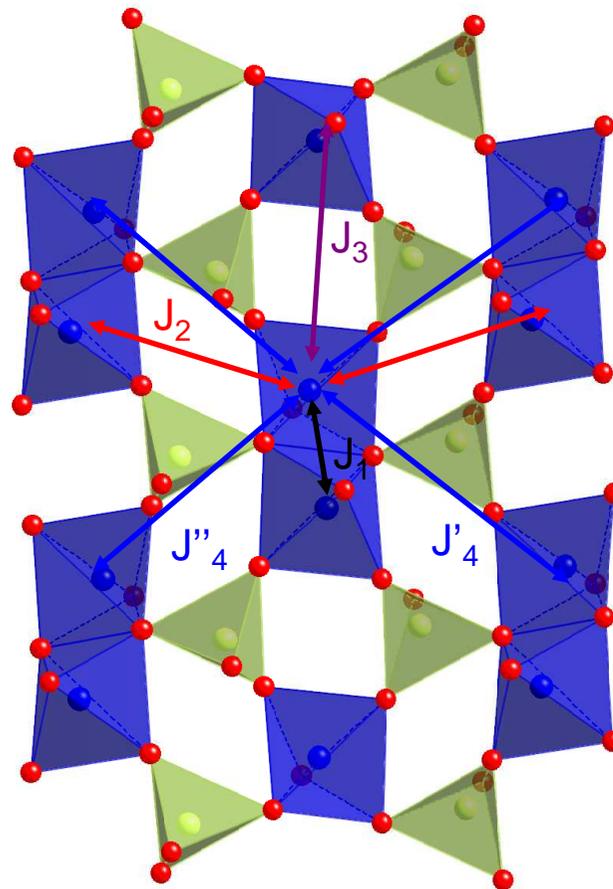
M. Isobe and Y. Ueda  
*J. Phys. Soc. Jap.* **65** (1996) 3142

➡ Isolated dimers :  $J = 146$  K

# Magnetic interactions in $\text{CsV}_2\text{O}_5$



$$\hat{H} = \hat{H}_0 + \sum_{i>j} J_{ij} \hat{\mathbf{S}}_i \hat{\mathbf{S}}_j$$



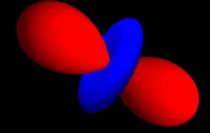
Magnetic couplings up to the fourth nearest neighbours :

- $J_1$  (3.073 Å) intradimer
- $J_2$  (5.386 Å) interchain
- $J_3$  (5.503 Å) intrachain
- $J'_4$  (5.950 Å) interchain
- $J''_4$  (6.653 Å) interchain

Simplified  $J_4$  interaction:

$$J_4 = (J'_4 + J''_4) / 2$$

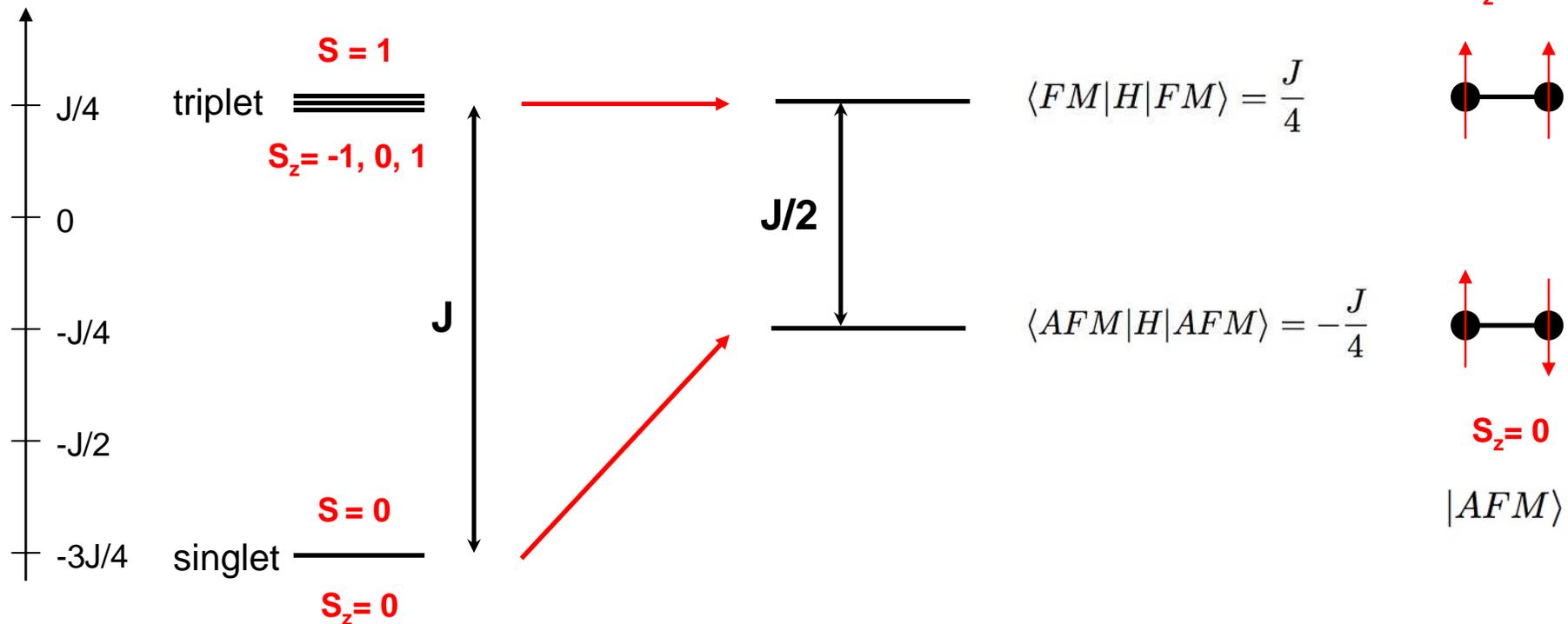
# Mapping DFT energies : single dimer



Estimation of  $J$  from DFT calculations?

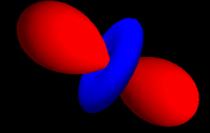
A system formed by a single spin-1/2 dimer  $\hat{H} = \hat{H}_0 + J\hat{S}_1\hat{S}_2$

Magnetic excitations written as a Heisenberg Hamiltonian



Spin contaminated nature of the AFM solution (not an eigenstate of  $S^2$ )

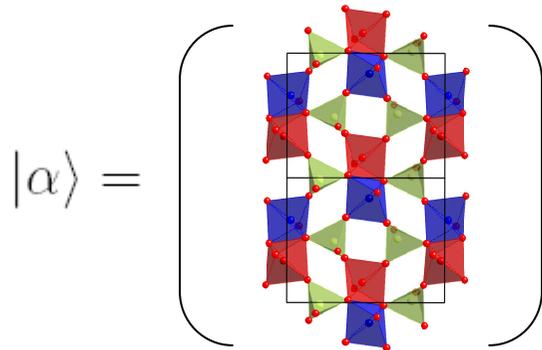
# Calculation of the exchange interactions



Heisenberg Hamiltonian for a general system

$$\hat{H} = \hat{H}_0 + \sum_{i>j} J_{ij} \hat{\mathbf{S}}_i \hat{\mathbf{S}}_j$$

Defining a DFT (collinear spin arrangement) state



A Kohn-Sham state (determinant) with a given projection of the spin in atom  $i$

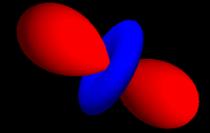
$$\hat{S}_{iz} |\alpha\rangle = \pm \frac{1}{2} |\alpha\rangle$$

The mean value of the Hamiltonian (the DFT total energy) :

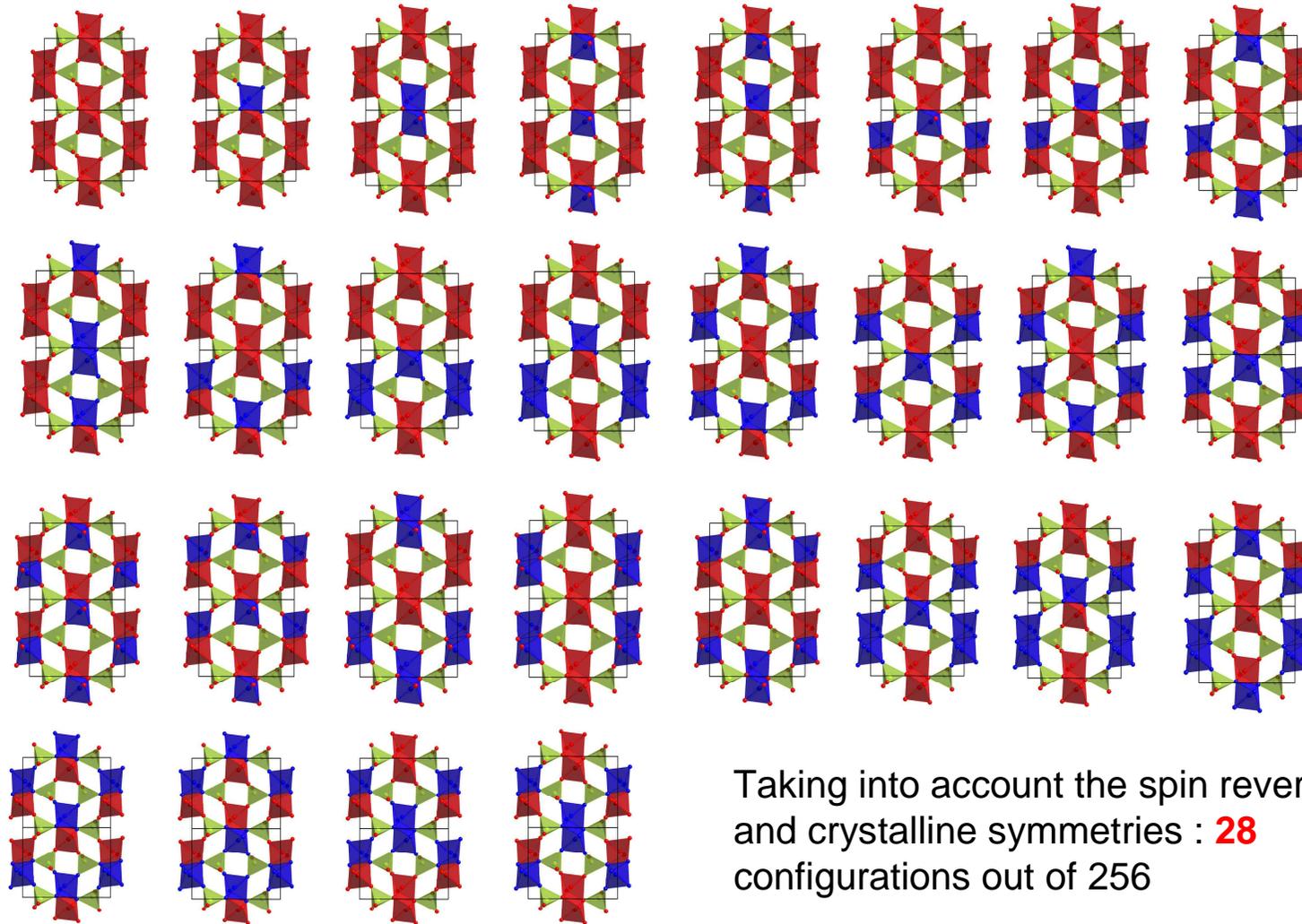
$$\epsilon_\alpha = \langle \alpha | \hat{H} | \alpha \rangle = \epsilon_0 + \sum_{i>j} \frac{J_{ij}}{4} \sigma_i \sigma_j \quad \sigma_i = \pm 1$$

can be written as an Ising Hamiltonian with the same  $J$ 's

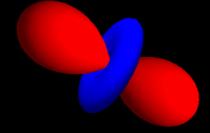
# Spin configurations



64-atom supercells (doubled along the chain direction z)

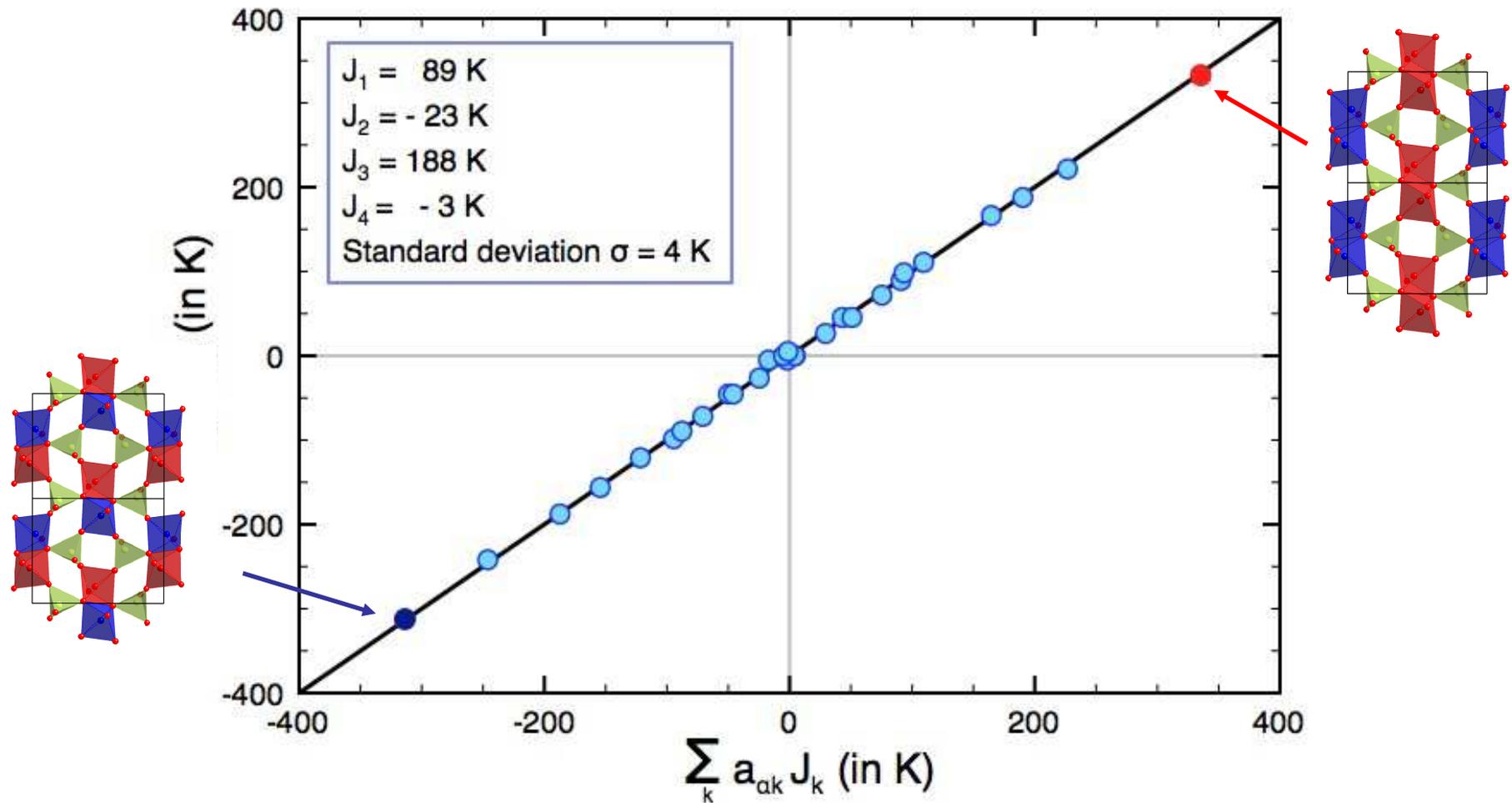


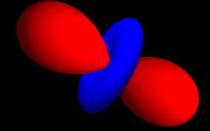
# Mapping of DFT energies: fitting procedure



Optimisation of the Ising parameters

$$F = \sum_{\alpha=1}^{28} g_{\alpha} \left( \epsilon_{\alpha} - \epsilon_0 - \sum_{i=1}^4 a_{i\alpha} J_i \right)^2$$

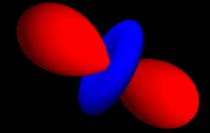




Ex. Int. (K)	$J_1$	$J_2$	$J_3$	$J_4$
PBE0	89	-23	188	-3
PBE	180	-54	378	-11

- Dominant AF interactions along the chain
- Third nearest- neighbor  $J_3 > J_1$
- Weak and ferromagnetic inter-chain interaction
- A semi-local functional GGA increases the amplitudes by  $\sim 2$  with respect to the exact exchange one

# Proposed superexchange paths



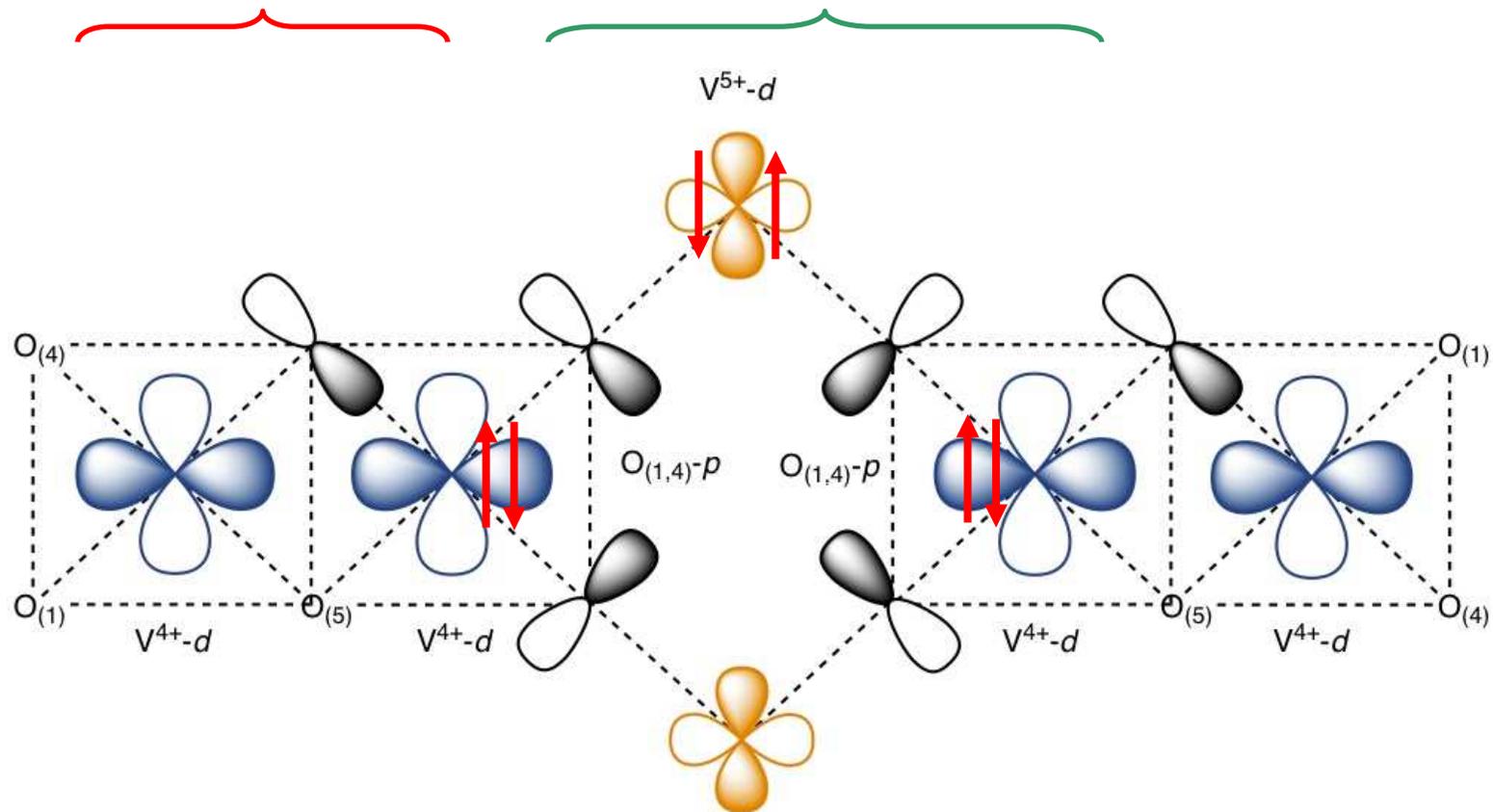
Motivated by the symmetry of the atomic orbitals

Close-to-orthogonal arrangement of the  $V^{4+}-d$  and  $O-p$  orbitals suggests a

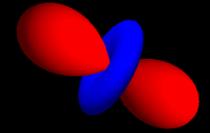
**Weak  $J_1$**

Non zero density of the  $V^{5+}-d$  orbital and allows a relatively

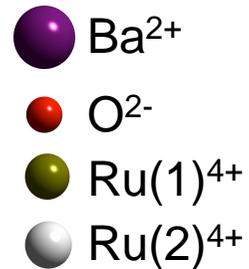
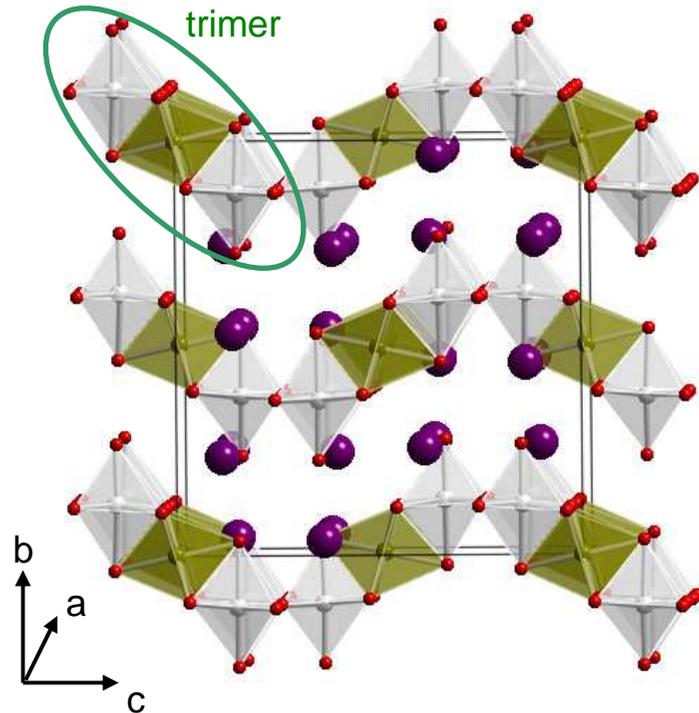
**Large  $J_3$**



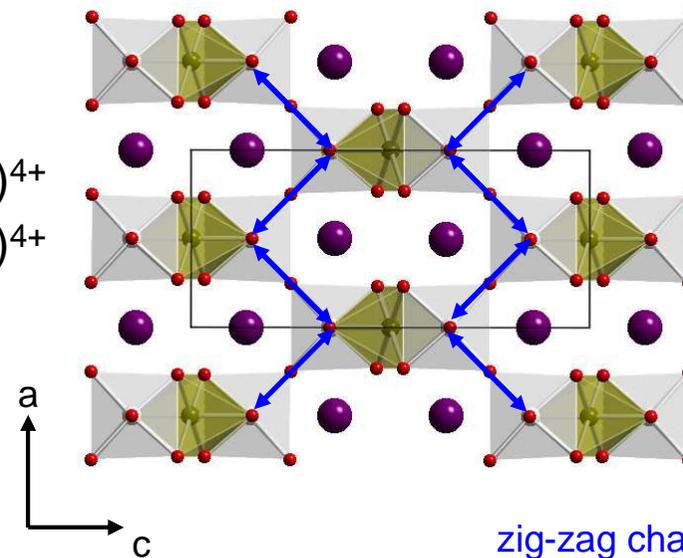
# A "not so localized" system : $\text{Ba}_4\text{Ru}_3\text{O}_{10}$



Orthorhombic structure  $Cmca$   
2D layers of  $\text{Ru}_3\text{O}_{12}$  trimers



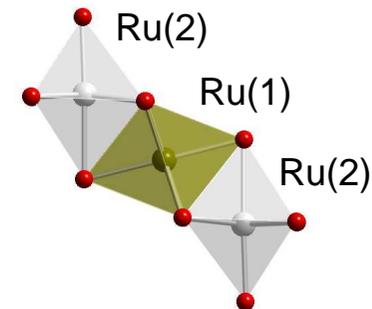
Structure of the layers



zig-zag chains along a

Face-shared  $\text{Ru}_3\text{O}_{12}$  trimers bring  
Ru ions very close ( $2.55\text{\AA}$ )

➡ possibility of metal-metal bonding



# Experimental results

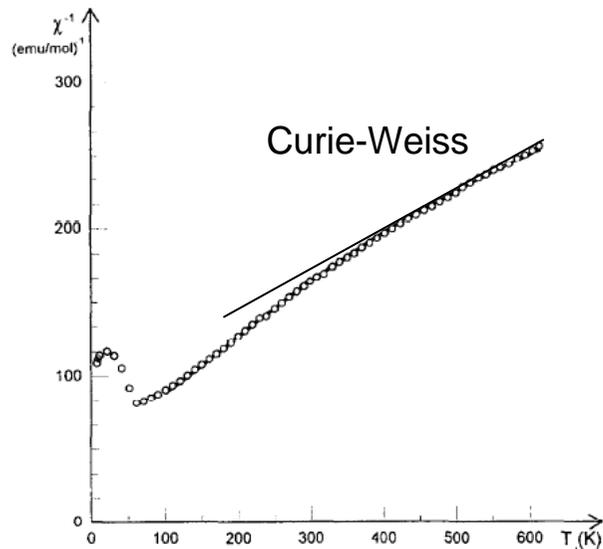
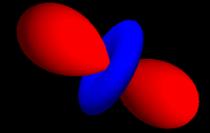
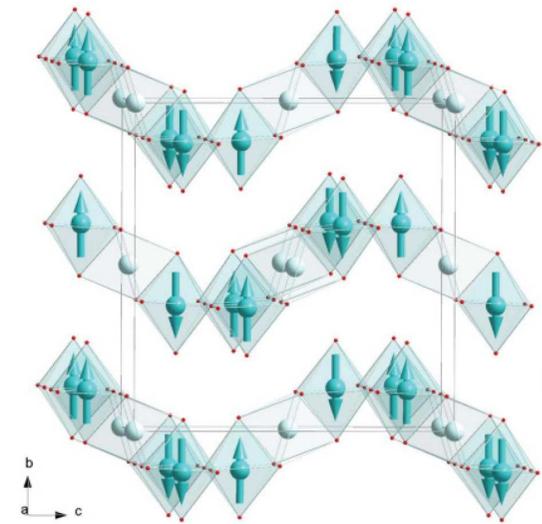
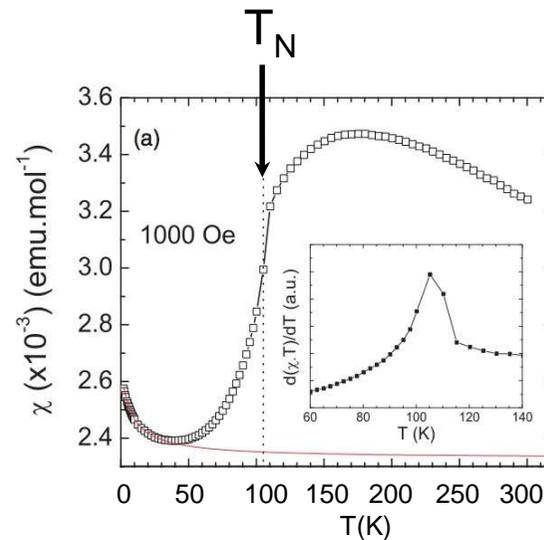


Fig. 6. Thermal variation of the reciprocal susceptibility of  $\text{Ba}_3\text{Ru}_3\text{O}_{12}$  (circles, observed; full line, calculated).

C. Dussarat *et al.*  
*J. All. Comp.* **233** (1996) 15

➡ Curie Constant consistent with three  $S=1$  (low-spin)  $\text{Ru}^{4+}$  per trimer @ high temperature

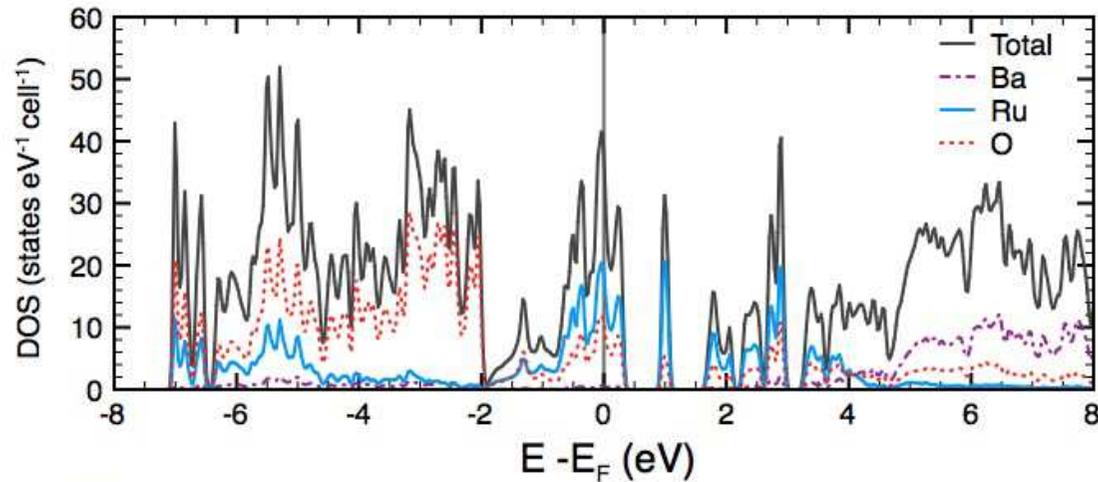
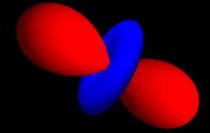


Y. Klein *et al.*  
*Phys. Rev. B* **84** (2011) 054439

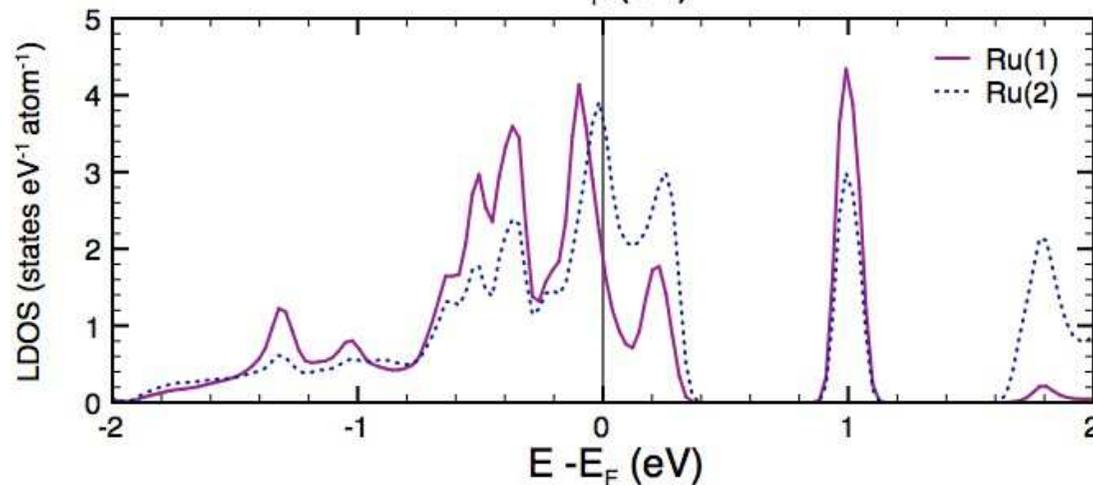
➡ AFM ground-state of a new kind. From neutron diffraction :

- Ru(2) show an AFM order
- Ru(1) have NO ordered moments

# Non-magnetic DOS



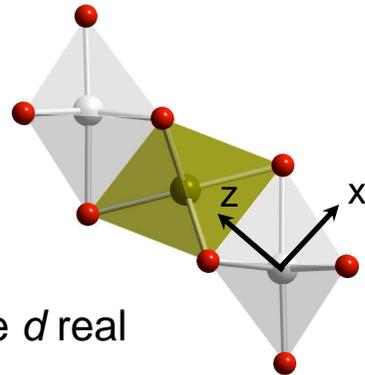
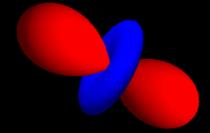
➔ Metallic  
Filling of 2/3 of the Ru- $t_{2g}$  orbitals :  
Ru(1)<sup>4+</sup> and Ru(2)<sup>4+</sup>



➔  $n(E_F) = 3.7 \text{ (eV atom}^{-1})$  for Ru(2)  
 $n(E_F) = 1.8 \text{ (eV atom}^{-1})$  for Ru(1)

➔ According to Stoner criterion, different behavior toward ferromagnetic instability

# Non-magnetic DOS



Alternative  $d$  real orbitals

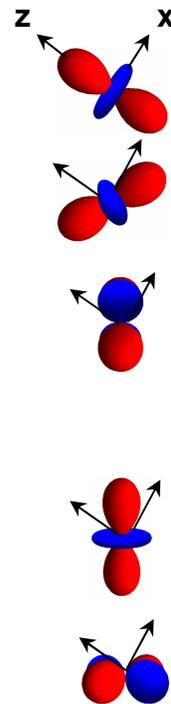
$$|t_{2g}^0\rangle = |d_{z^2}\rangle$$

$$|t_{2g}^+\rangle = \sqrt{\frac{2}{3}}|d_{x^2-y^2}\rangle - \sqrt{\frac{1}{3}}|d_{xz}\rangle$$

$$|t_{2g}^-\rangle = \sqrt{\frac{2}{3}}|d_{xy}\rangle + \sqrt{\frac{1}{3}}|d_{yz}\rangle$$

$$|e_g^+\rangle = \sqrt{\frac{1}{3}}|d_{x^2-y^2}\rangle + \sqrt{\frac{2}{3}}|d_{xz}\rangle$$

$$|e_g^-\rangle = \sqrt{\frac{1}{3}}|d_{xy}\rangle - \sqrt{\frac{2}{3}}|d_{yz}\rangle$$

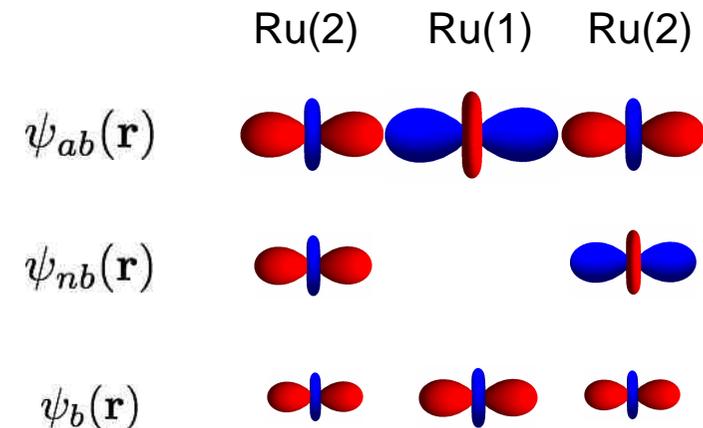


We expect  $\sigma$ -bonding between  $|t_{2g}^0\rangle$  states to form bonding, non-bonding and anti-bonding MOs:

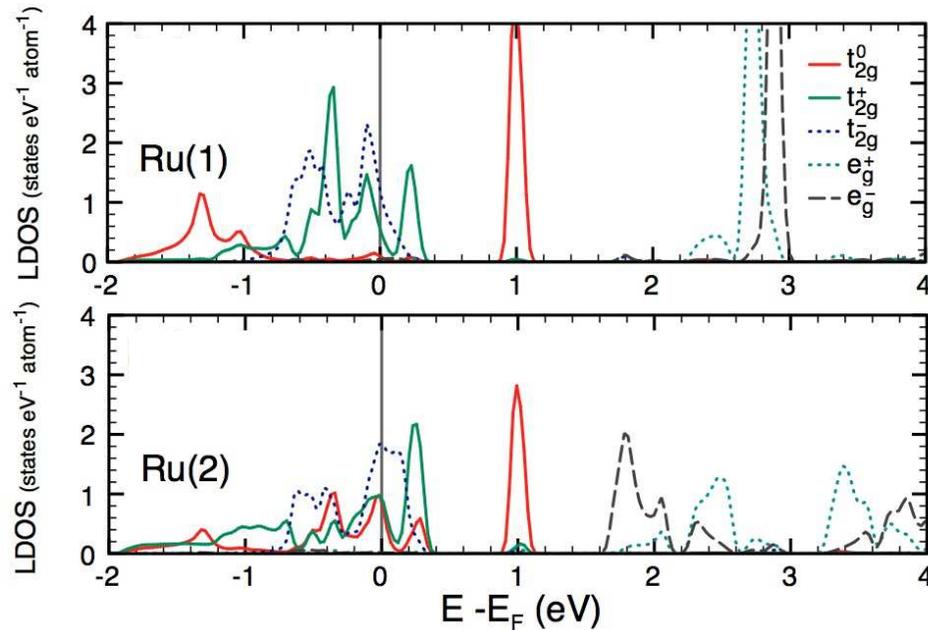
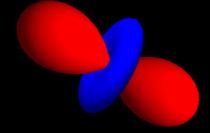
$$|\psi_b\rangle = \frac{1}{\sqrt{4(1+\sqrt{2}S)}} (|\phi'_{(2)}\rangle + \sqrt{2}|\phi_{(1)}\rangle + |\phi''_{(2)}\rangle)$$

$$|\psi_{nb}\rangle = \frac{1}{\sqrt{2}} (|\phi'_{(2)}\rangle - |\phi''_{(2)}\rangle)$$

$$|\psi_{ab}\rangle = \frac{1}{\sqrt{4(1-\sqrt{2}S)}} (|\phi'_{(2)}\rangle - \sqrt{2}|\phi_{(1)}\rangle + |\phi''_{(2)}\rangle)$$



# Non-magnetic DOS



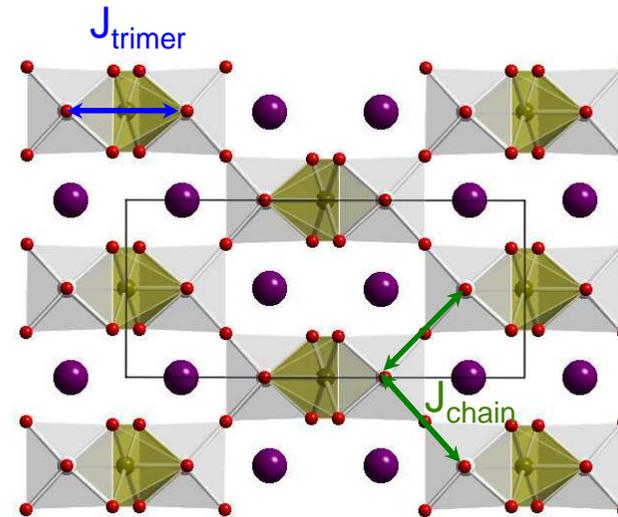
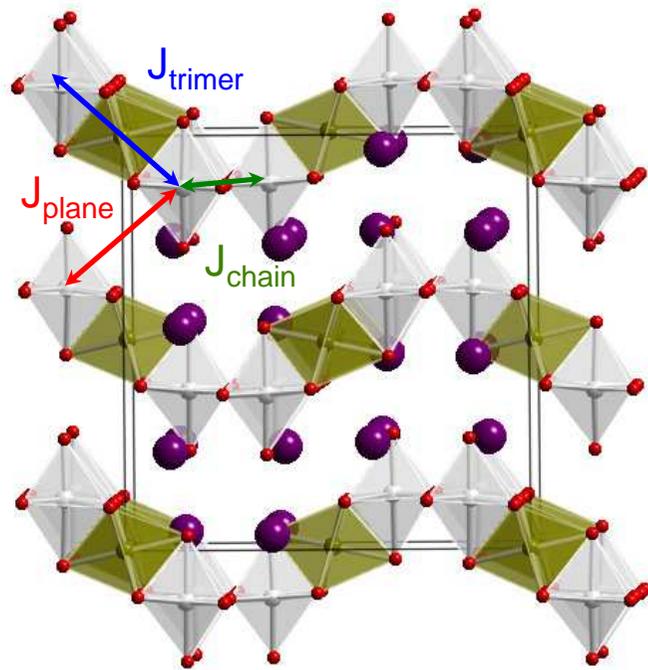
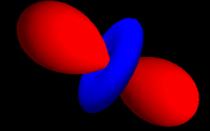
- ➔  $|t_{2g}^0\rangle$  are present at  $E_F$  for the Ru(2) but not for the Ru(1) : effect of metal-metal bonding
- ➔ The non-bonding MO has no Ru(1) component at  $E_F$

Ferromagnetic calculations confirm the weaker polarization of Ru(1):

M ( $\mu_B$ )	Ru(1)	Ru(2)
GGA	0.34	0.94
GGA+U	0.09	1.12

} reduced from  $2 \mu_B$  expected for  $S=1$  due to covalency

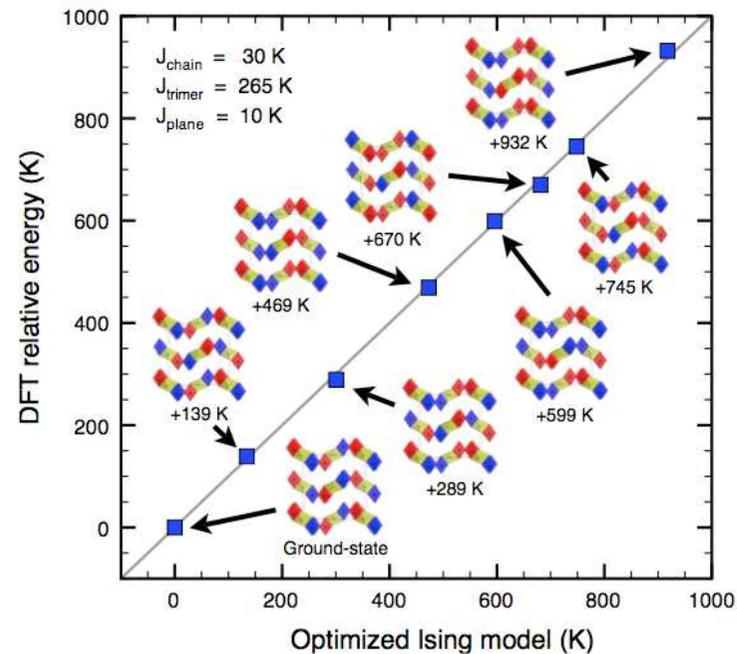
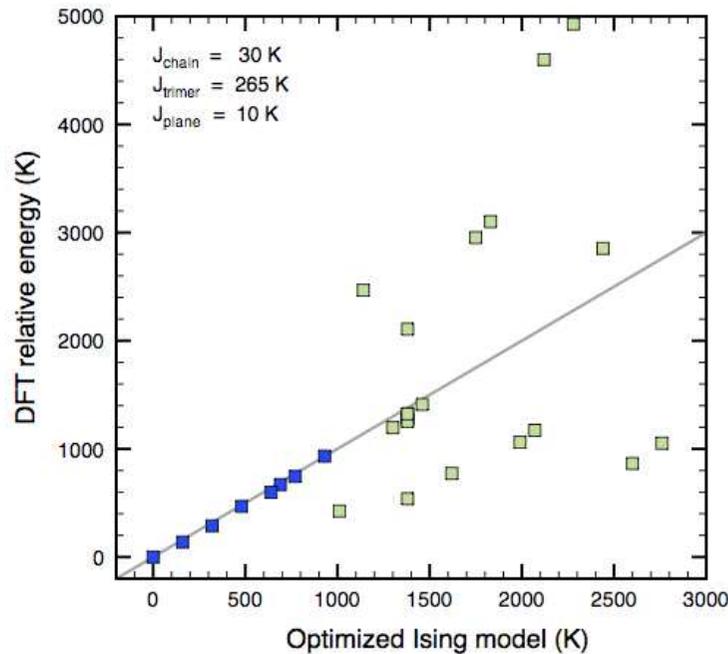
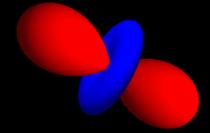
# Definition of the couplings



Mean value of the Heisenberg Hamiltonian on the DFT states ( $S=1$  on Ru(2)) :

$$\epsilon_\alpha = \langle \alpha | \hat{H} | \alpha \rangle = \epsilon_0 + \sum_{i>j} J_{ij} \sigma_i \sigma_j \quad \text{with} \quad \sigma_i = \pm 1$$

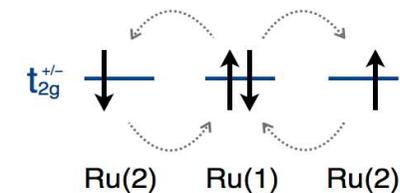
# Results : GGA and GGA+U



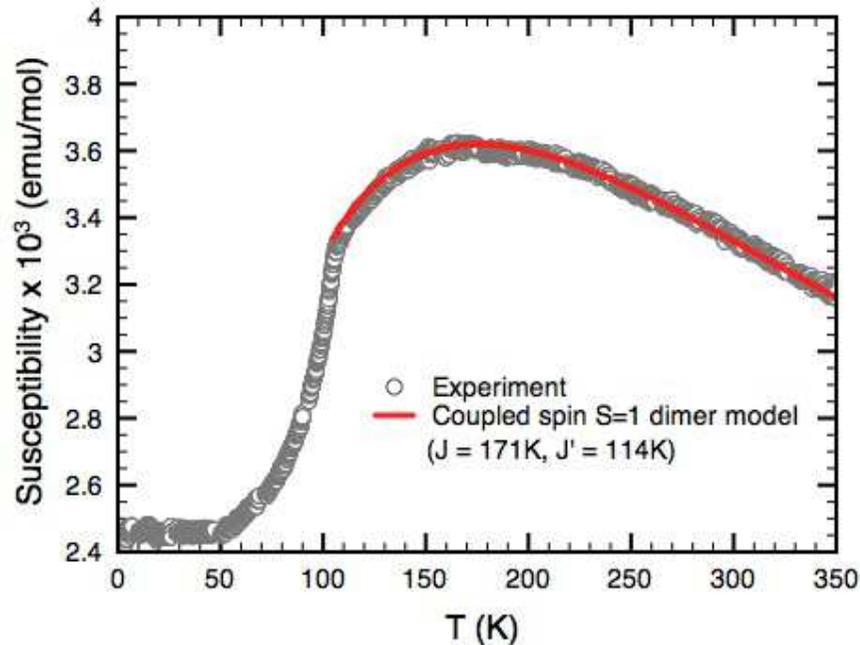
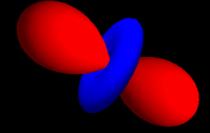
- ➡ No single set of  $J$ 's can fit the entire spectrum of calculated magnetic excitations
- ➡ A subset of low energy configurations can be used instead

Ex. Int. (K)	$J_{\text{trimer}}$	$J_{\text{chain}}$	$J_{\text{plane}}$
GGA	344	86	25
GGA+U	265	30	10

Kinetic super-exchange mediated by Ru(1) :



# What about the susceptibility ?



➔ Based on DFT results, the susceptibility is fit with a model of coupled (mean-field) spin S=1 dimers hold by Ru(2) :

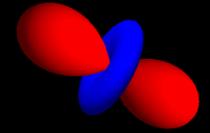
$$\chi = \frac{\chi_{\text{dim}}}{(1 + \lambda\chi_{\text{dim}})} + \chi_0$$

with :  $\lambda = \frac{J_{\text{inter}}}{N_A(g\mu_B)^2}$

$$J_{\text{inter}} \approx 2J_{\text{chain}} + 2J_{\text{plane}}$$

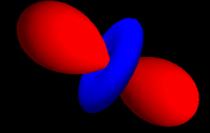
$$\chi_{\text{dim}} = \frac{2\beta g^2 \mu_B^2 N_A [1 + 5 \exp(-2\beta J_{\text{trimer}})]}{[3 + \exp(\beta J_{\text{trimer}}) + 5 \exp(-2\beta J_{\text{trimer}})]}$$

➔ Confirms the picture of strong AFM dimers coupled by weaker AFM inter-dimer interactions stabilizing a long-range Néel order.



Thank you !

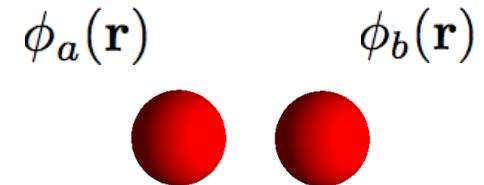
# Breakdown of localized spin models ?



For a two-site, two-electron system :

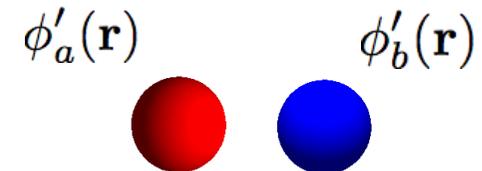
The WF calculated in DFT for a FM spin arrangement

$$\psi^{\text{KS}}(S_Z = 1) = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_a(\mathbf{r}_1)\alpha(\sigma_1) & \phi_b(\mathbf{r}_1)\alpha(\sigma_1) \\ \phi_a(\mathbf{r}_2)\alpha(\sigma_2) & \phi_b(\mathbf{r}_2)\alpha(\sigma_2) \end{vmatrix}$$



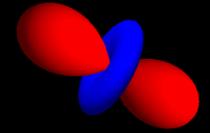
The WF calculated in DFT for a AFM spin arrangement

$$\psi^{\text{KS}}(S_Z = 0) = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi'_a(\mathbf{r}_1)\alpha(\sigma_1) & \phi'_b(\mathbf{r}_1)\beta(\sigma_1) \\ \phi'_a(\mathbf{r}_2)\alpha(\sigma_2) & \phi'_b(\mathbf{r}_2)\beta(\sigma_2) \end{vmatrix}$$

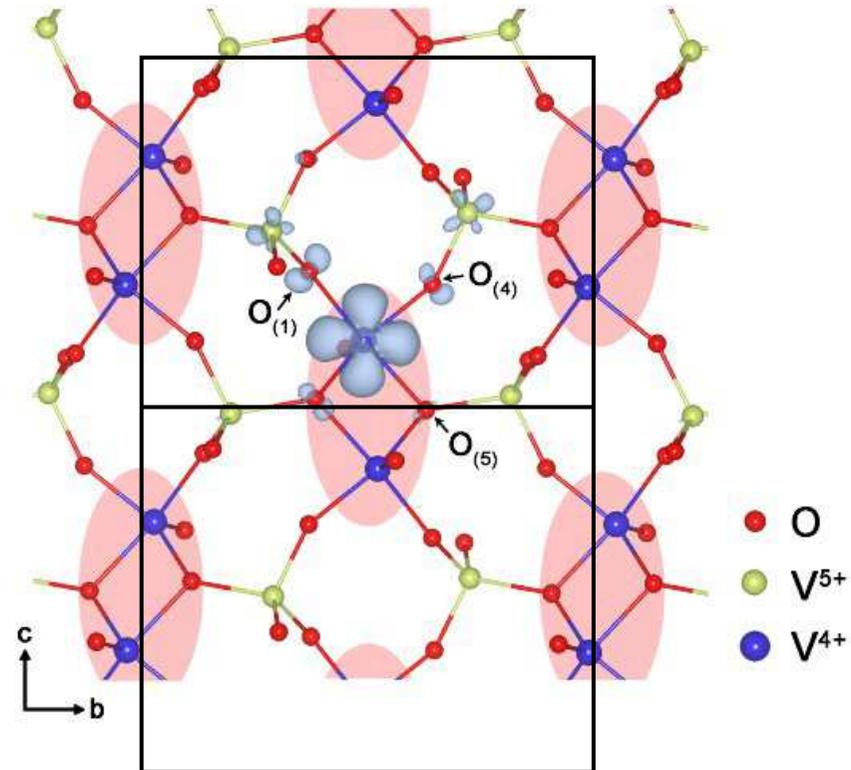
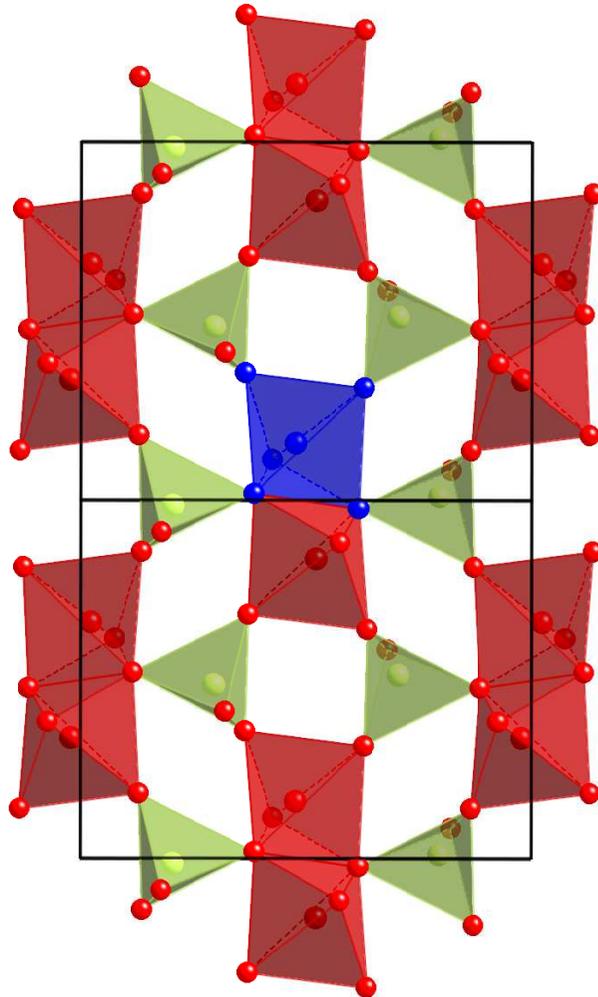


➡ The assumption that the spatial WF does not depend on the magnetic order is questionable for strongly covalent systems

# Orbitals participating to the superexchange



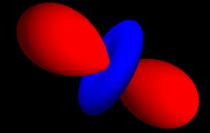
Electron density associated to a single  $V^{4+}$  ion : the only one in a flipped spin



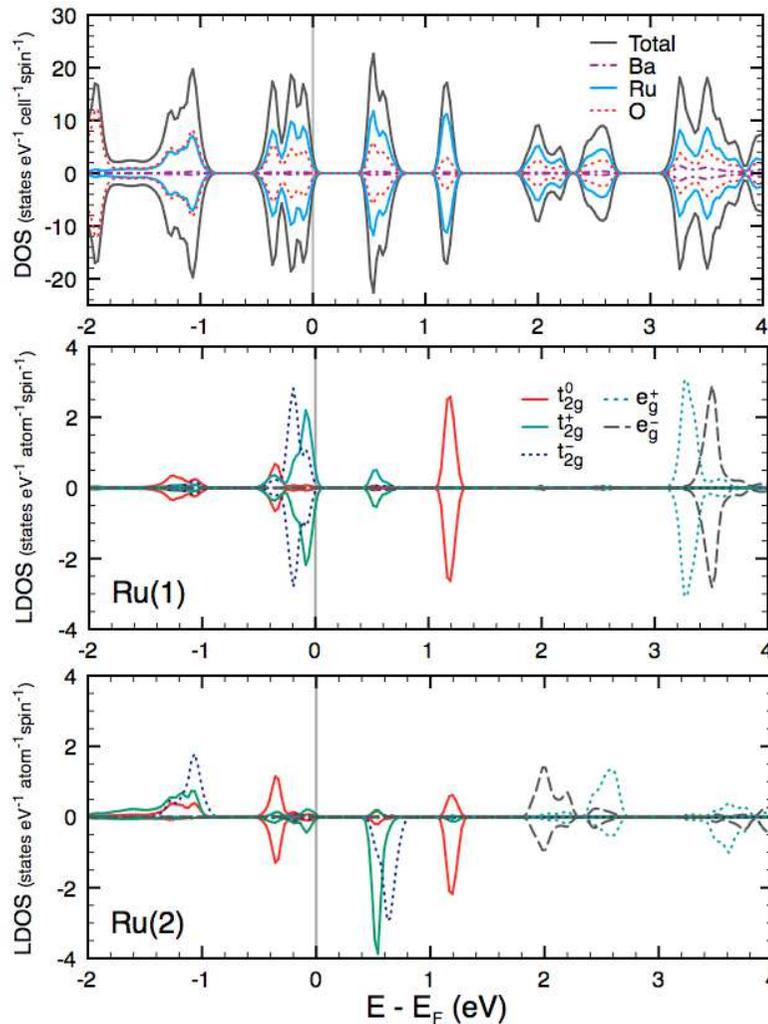
The magnetic orbital presents :

- Dominant  $d_{y^2-z^2}$  orbital on the  $V^{4+}$  ion
- Specific p symmetry on the neighboring O
- Non zero density on the  $V^{5+}$  ion

# The AFM ground-state



Both GGA and GGA+U predict the same AFM (exp.) ground-state order



➔ Ru(2) hold a spin  $S=1$  through the polarization of  $t_{2g}^+$  and  $t_{2g}^-$  states

Ru(1) is paramagnetic through a double occupation of both  $t_{2g}^+$  and  $t_{2g}^-$  states

➔ Kinetic superexchange mechanism mediated by paramagnetic Ru(1) :

